Mechanism Performance under Strategy Advice and Sub-Optimal Play:
A School Choice Experiment

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Abstract

We implement a laboratory experiment to study how strategy advice affects participants' decisions in a school choice game. In the strategy-proof Deferred Acceptance (DA) mechanism, strategy advice prompts more participants to choose the dominant strategy of truth-telling. In the Immediate Acceptance (IA) mechanism, strategy advice to implement one of two heuristic strategies that are widely recommended in the field induces participants to choose one of those strategies. We then use our data to perform exploratory analyses on how the variation in the proportion of participants who choose sub-optimal strategies affects mechanism performance. Consistent with the literature, we find that DA outperforms IA in standard efficiency and stability tests, increasingly so in our advice treatment. We next implement an analysis of individual welfare using a new partially-ordered typology of DA strategies. DA outperforms IA, particularly under sub-optimal play, for almost any individual’s welfare.

Keywords: school choice, experiment, strategy advice, mechanism design, sub-optimal play

JEL Codes: C78 (Bargaining Theory; Matching Theory), C92 (Laboratory, Group Behavior), and D82 (Asymmetric and Private Information; Mechanism Design)

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1 Introduction

Policymakers increasingly turn to school choice mechanisms to break the link between a student’s residence and their public school assignment. Yet, the choice between the two most popular mechanisms remains fraught for the public as well as economists. The theoretical and experimental literatures have weighed in on both sides, but one issue that repeatedly arises is that mechanism performance largely depends on participants’ strategy choices. People do not always play a theoretically optimal strategy for a number of reasons. One reason is that determining the optimal strategy may be costly. Real-world school choice markets address this issue through strategy advice. The school system, newspapers, and even internet blogs provide strategy advice intended to inform parents’ strategy choices. Despite the prevalence of strategy advice in school choice, little work considers how structured advice affects participants’ strategy choices. We address this gap in the literature with a laboratory experiment. We then exploit the induced variation in participant strategy from the strategy advice to address the ongoing debate about relative mechanism performance under sub-optimal strategies.

We replicate the seminal school choice laboratory experiment of Chen and Sonmez (2006), then add a strategy advice treatment, to study how strategic behavior responds to the advice. Under both the strategy-proof Deferred Acceptance (DA) mechanism and the manipulable Immediate Acceptance (IA) mechanism, our strategy advice substantially increases the proportion of participants who choose a recommended strategy. Next, we do exploratory analyses on the relative performance of the DA and IA mechanisms. In line with other studies (e.g. Chen and Sonmez, 2006; Calsamiglia et al., 2010), DA is both more efficient and more stable than IA in our experiment. This difference in performance is exacerbated in the advice treatments of our experiment.

Our new approach to evaluating relative mechanism performance considers individual welfare. We ask the following question: would a given participant in our experiment have ex-ante preferred to participate in a DA or IA session? To answer this question, we use recombinations of a tie breaker and the strategies of other participants to estimate the expected payoffs to participants who chose a recommended strategy in an IA session based on their observed strategy. Next, we again use this recombinant estimation technique to estimate the participant’s expected payoff in a DA session. Because the participant did not actually participate in a DA session, this latter estimation requires the determination of a counter-factual strategy for the participant in DA. We address this requirement by developing a new typology of DA strategies that partially dominance-orders all possible strategies. This typology provides the first ordered classification of sub-optimal strategies in DA.

Comparing a participant’s expected payoff from their observed strategy in IA to that same participant’s expected payoffs from playing a series of increasingly irrational strategies in DA, we find that almost any participant who followed our strategy advice would have preferred to take part in a DA session rather than an IA session. On the other hand, some participants who do not follow the advice to implement one of two heuristic strategies fare better in IA sessions. In particular, the most disadvantaged participants, determined by their district school’s preference ranking, who do not follow our advice choose strategies that generate higher payoffs than playing the dominant strategy in DA. In summary, we show that the DA mechanism is not only more efficient in terms of overall welfare, but is welfare-enhancing for almost any individual, particularly those who play heuristic strategies like the ones recommended in real-world markets. The exceptions to this conclusion are disadvantaged participants who play more sophisticated strategies than the simple

\[^1\text{Participants with the lowest district school rank would be unwise to play one of our recommended strategies, which are commonly observed in the field.}\]
heuristics, who constitute a relatively small proportion in our sample.

**The Experiment**

To test how participants respond to strategy advice, we design our experiment as a variant of the school choice laboratory experiment in Chen and Sonmez (2006). Chen and Sonmez test the truth-telling rates and efficiency of three school choice mechanisms: DA, IA, and Top Trading Cycles (TTC). We restrict our attention to DA and IA, then add a strategy advice treatment for each mechanism. Chen and Sonmez’s rich experimental design allows us to study participants’ strategy choices in a relatively elaborate environment intended to replicate the size and complexity of real-world school choice problems.

The strategy advice that we give is inspired by strategy advice observed in the field. For example, the Minneapolis Central Placement and Assessment Center (CPAC) recommends that parents not tell the truth in the IA assignment mechanism used in that city. Instead, CPAC recommends that parents list their most preferred school first, but advises them to rank their district school second or third, even if this school is not their true second or third choice. We call this our "Risky" strategy. News sources also contribute advice. C. Mas in The Seattle Press (1998) suggests that parents may want to list their district school first in the IA mechanism to avoid being placed at a less preferred school. We call this our "Safe" strategy. These two simple heuristic strategies are also observed in lab experiments (e.g. Chen and Sonmez, 2006), though they are unlikely to be optimal for many students.

Strategy advice in DA is more straightforward because there is only one correct recommendation: tell the truth by submitting preferences in order from most- to least-preferred school. School districts that adopt DA, like the New York City Department of Education, distribute manuals that include strategy advice to tell the truth when ranking schools.

We find that, in both IA and DA, our strategy advice increases the proportion of participants who choose a recommended strategy. In DA, advice to play the dominant strategy of truth-telling increases the proportion of participants who truthfully reveal their preferences from 31% to 50%. Our strategy advice in IA, inspired by strategy advice from the field, increases the probability that a participant chooses one of the two simple heuristic strategies from 51% to 71%. Overall, we provide strong evidence that strategy advice has an economically large and statistically significant effect on participants’ strategy choices in the lab.

We provide the first evidence on how strategy advice affects behavior in a non-strategy-proof mechanism and new evidence on strategy-proof mechanisms in a more complex environment. Independently of our study, Ding and Schotter (forthcoming) study strategy advice that is devised and shared by participants in the same two mechanisms as our experiment. In their experiment, participants in one generation pass down a recommended ordering over three schools to the next generation. In contrast, our strategy advice is consistent across all participants, in line with the type of advice that may be distributed by school districts for example. This difference appears to matter since Ding and Schotter find an increasing rate of truth-telling in the IA mechanism with strategy advice over generations. In our experiment, truth-telling rates in IA drop significantly in the strategy advice treatment. Similarly for the DA mechanism, while strategy advice derived by participants in Ding and Schotter decreases the truth-telling rates, structured strategy advice in our experiment leads to significantly higher rates of truth-telling. The contrasting results of our studies suggest that further research is needed since participants in real world school

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2Truth-telling is the weakly dominant strategy in the DA mechanism because it is strategy-proof. DA fails to be strategy-proof in some circumstances. Specifically, when the number of schools students can report is less than the total number of schools the student prefers to their outside option or when there are fewer available seats at a student's district school than there are students assigned to that school, truth-telling may not be the dominant strategy.
choice mechanisms likely receive both structured advice like ours and advice from their network, as in Ding and Schotter.

Zhu (2015) also considers strategy advice that is passed from participant to participant in the DA mechanism. In her lab experiment, Zhu has participants in one session play the mechanism repeatedly, then asks them to provide strategy advice to participants of the same type (preferences and priorities) in future sessions. The strategy advice takes the form of a recommended ranking over the three schools and, due to the learning that occurs over 15 rounds, is mostly correct. She finds that advice increases truth-telling rates, but not at a statistically significant level. We show that strategy advice increases truth-telling in DA at a statistically significant level in an environment with greater complexity (i.e., more players types and choices).

Two papers by Guillen and co-authors study strategy advice on the TTC mechanism that is structured, like our own, to be the same for all participants. First, in an introductory microeconomics course at the University of Sydney, Guillen and Hakimov (2017) test the effect of different information environments on participant strategy. In their experiment, the authors explain the strategy-proofness property of TTC in treatment one, explain just the mechanism’s algorithm in treatment two, and explain both the property and algorithm in treatment three. They find that telling participants the mechanism is strategy-proof increases the likelihood that students truthfully reveal their top preference out of three possible topics for their term paper. Compared to our experiment, the environment in Guillen and Hakimov has fewer choices, perfectly correlated preferences, and the observation of truth-telling limited to the top-ranked choice, but we find comparable results for the DA mechanism.

Guillen and Hing (2014) evaluate the effects of correct versus incorrect third-party advice on strategic behavior in TTC. The correct advice advocates ranking schools according to the participant’s true preferences while the incorrect advice suggests ranking the participant’s district school first. Whether correct or incorrect, strategy advice from a third-party decreases the likelihood that participants truthfully reveal their preferences compared to a baseline treatment of no advice. Our strategy advice comes from an authority figure (the experimenter) and positively affects the rate of truth-telling, suggesting a need for further research into the effect of the source of advice.

Two other papers indirectly consider strategy advice. Ding and Schotter (2015) study chat between participants in a laboratory experiment, where the content of the chat could be considered strategy advice from other participants (but could also be signaling information about other participants’ preferences, priorities, and strategies). The "chat" treatment induces more participants to change their strategy after the chat, particularly when chatting with participants who have the same preferences and priorities as them. Chat also increases the number of stable outcomes in both the DA and IA mechanisms. Individual payoffs increase only when participants communicate with others that are unlike them. Braun et al. (2014) incorporate strategy "coaching" in their lab experiment. Although they do not study strategy advice, the authors show that strategy coaching increases the likelihood of truth-telling in a strategy-proof mechanism with a small control group. In the mechanism that rewards preference manipulation, they find a lower proportion of participants play the naïve strategy of truth-telling. Our strategy advice is structured the same for each participant, as opposed to strategy coaching, and their experiment incorporates learning over rounds, while we study a one-shot game that mimics real world school choice applications.

**Mechanism Performance**

We use our experimental data to do exploratory analyses on the relative performance of DA and IA under varying levels of sub-optimal behavior. These analyses contribute to the
literature launched by Abdulkadiroğlu and Sonmez’s (2003) theoretical paper and Chen and Sönmez’s (2006) laboratory experiment pitting DA against IA. Their papers catalyzed changes in a number of school systems, including the well-known conversion from IA to DA by the Boston school system in 2005 (Abdulkadiroğlu et al., 2006). Foremost of the school board’s reasons for switching mechanisms were the strategy-proofness and stability of DA. Strategy-proofness theoretically eliminates the cost of strategizing and levels the playing field for students with heterogeneous costs to strategizing. The stability of DA is appealing because school boards believe the elimination of justified envy gives the appearance of a more "fair" outcome (Abdulkadiroğlu et al., 2006). The trade-off for the strategy-proofness and stability of DA, however, is IA’s theoretically possible greater efficiency.

DA is not Pareto efficient, but theoretically it generates the most efficient stable solution. DA’s efficiency and stability, however, rely on participants choosing the dominant strategy of truth-telling. Lab experiments find truth-telling rates far from 100% (e.g. Chen and Sönmez, 2006; Klijn et al., 2013; Basteck and Mantovani, 2018). Empirical studies also find preference manipulation in DA (e.g. Hassidim et al., 2016), confirming that the sub-optimal rates of truth-telling are not an artifact of the lab. Rees-Jones (2018) studies the high-stakes residency match that implements the DA mechanism. Among medical students with prevalent (and correct) strategy advice in a system that has had years to establish the strategy-proofness of DA, Rees-Jones still finds students that fail to play the dominant strategy of truth-telling. We also find high proportions of participants choosing sub-optimal strategies in our experiment, even in the advice treatment when participants are advised to choose the dominant strategy of truth-telling.

Theoretical work argues that IA’s greater efficiency may outweigh DA’s stability and strategy-proofness, particularly when students have limited information about each others’ preferences and about schools’ priorities (Miralles, 2009; Abdulkadiroğlu et al, 2011; Troyan, 2012). Miralles (2009) and Abdulkadiroğlu, Che, and Yasuda (2011) assert that IA may be more efficient than DA in practice because IA incorporates information about cardinal preferences in its algorithm. The strategic manipulability of IA allows participants to signal preference intensity, while DA only accounts for ordinal preferences. Troyan (2012) formalizes and generalizes Abdulkadiroğlu et al.’s argument to show that theoretically IA ex-ante Pareto dominates all strategy-proof mechanisms. The ex-ante argument hinges critically on no information being known—participants do not know their preferences, schools do not have known priorities, and the number of students at each priority level is unknown.

Arguments for the greater efficiency of IA rely on either all participants truth-telling or all playing strategies that correspond to the same sophisticated Bayesian-Nash equilibrium; however, experiments such as Featherstone and Niederle (2016) show that participants typically fail to play a non-truth-telling Bayesian-Nash equilibrium in IA. Bayesian-Nash equilibria are particularly unlikely if participants rely on simple heuristic strategies, which naturally occur in experiments like Chen and Sönmez (2006). While a simple heuristic strategy may be optimal for some people, in an environment with any sort of complexity, such strategies are generally sub-optimal and are unlikely to lead to a Bayesian-Nash equilibrium. It is important to note, however, that we cannot say for certain that the simple heuristic strategies we recommend are sub-optimal. Instead, we base our analysis only on the assumption that the induced variation in strategies led to varying proportions of sub-optimal strategies without distinguishing whether the advice or no advice treatment has the higher proportion.

Since experimental and empirical studies find that participants are choosing sub-optimal strategies, we consider how those strategies affect the relative performance of the DA and

\[3\text{An "optimal" strategy in IA is based on a participants’ beliefs about the choices of other participants, which we do not observe.}\]
IA mechanisms. Concerning the observed preference misrepresentation in the strategy-proof medical residency match, Rees-Jones (2018) writes:

"...the persistence of suboptimal behavior in this setting, even at low rates, suggests the requisite levels of intelligence, information, and incentivization needed to ensure full compliance may never be achieved in practice. Some strategic misunderstanding may be unavoidable in these settings, necessitating attention to the comparative performance of mechanisms in the presence of suboptimal behavior..."

We find that, consistent with the experimental literature (Chen and Sönmez, 2006; Calsamiglia et al., 2010), DA outperforms IA in both our advice and no advice treatments. Our strategy advice has little effect on the efficiency of DA; however, the advice to play a heuristic strategy in IA decreases efficiency. Likewise, stability increases under strategy advice in DA, but decreases in IA. Our experimental results show that sub-optimal play decreases the performance of DA, but DA still outperforms IA.

We next develop a new dimension to evaluating relative mechanism performance by considering the welfare of individual participants. Studying individual welfare allows us to observe heterogeneity in mechanism performance with respect to where a student’s district school ranks in their preferences. Specifically, we compare the welfare differences between participating in the IA versus the DA mechanism by the preference ranking of a participant’s district school. Participants with district schools that lead to a high payoff are at a relative advantage in our experiment, much as students who live in wealthy neighborhoods with good schools in real-world school choice markets have an advantage.

To conduct this analysis, we evaluate individual participants’ welfare in a counterfactual exercise using recombinant estimations of expected payoffs. For each participant in an IA treatment who chose one of the recommended heuristic strategies, we estimate the expected (rather than observed) payoff to their observed strategy and the expected payoff from participating in a DA treatment instead. We do not, however, know the counter-factual strategy a participant would have chosen in a DA treatment. Assuming the participant would choose the dominant strategy exaggerates the payoffs to DA (particularly considering we observe that less than half the participants play the dominant strategy).

We address this issue by developing a new typology of DA strategies that allows us to partially order sub-optimal strategies in DA by dominance. Sub-optimal strategies in DA are currently classified by biases identified and described in Chen and Sönmez (2006). While these biases are informative for understanding sub-optimal behavior, they do not allow us to rank strategies according to their dominance. The typology of DA strategies we develop addresses this issue by deconstructing DA strategies into three strategic aspects. A participant who understands all three strategic aspects of DA chooses a dominant strategy. Participants who understand more — but not necessarily all — of these strategic aspects play "more rational" strategies compared to participants who understand fewer of them.

Using this typology, we calculate the expected payoff for each participant in the IA sessions of, instead, participating in a DA session using increasingly irrational strategies as counter-factual strategies. We find that almost any participant choosing one of the recommended heuristic strategies in the IA treatment would have preferred to participate in the DA treatment. Participants can choose strategies that are "far" from the dominant strategy in DA and still be better off than by playing a heuristic strategy in IA. The results are mixed for participants who do not choose one of the recommended strategies in IA. Participants who are at a particular disadvantage in the school choice game (defined by their district school’s ranking in their preferences) would be considerably better off playing

\footnote{We say "a" as opposed to "the" here to emphasize that there are multiple dominant strategies since schools ranked below a participant’s district school are irrelevant.}
any strategy in DA if they are the type to follow strategy advice, while disadvantaged participants who choose a non-heuristic strategy fare better in the IA mechanism compared to even the dominant strategy in DA.

**Organization**

The remainder of this paper is organized as follows. Section two presents the formal school choice problem, the two mechanisms, and their theoretical properties. Section three describes our experimental design and treatments to test the effects of strategy advice on participants’ strategic decisions. We discuss the lab results in section four. In section five, we comprehensively evaluate the relative performance of the two mechanisms under sub-optimal strategy. First, we evaluate the relative efficiency and stability of IA and DA under varying levels of sub-optimal play. Then, we introduce our new typology of DA strategies that allows us to evaluate the welfare of participants who choose one of the recommended heuristic strategies. Finally, we conduct the counter-factual analyses of individual welfare. Section six concludes with a discussion of our results.

**2 The Theoretical School Choice Problem**

We set up the school choice problem in this section to discuss the relevant properties of mechanisms that assign students to schools in a one-sided matching game. These properties are important to understand because they inform our mechanism performance analysis as well as the strategy advice we give to participants. We then detail the algorithms for the two mechanisms we study in this paper, DA and IA, and their properties.

The school choice problem is a one-sided matching game in which a set of $N$ students are matched to $M$ schools with limited capacities $q_m$ for $m \in M$. Students have a strict preference ordering $P_i$ over schools and schools have weak priority levels $F_m$ over students. Priorities are coarser than preferences, allowing for indifferences, so a fair lottery is used to break ties between students. These priority levels are fundamentally different from student preferences because schools are objects to be consumed by the students. In real-world settings, priorities are set by education boards or state laws. For example, priority is often given to a student whose sibling already attends the school or to a student who lives within walking distance of the school.

A solution to the school choice problem is a matching $\mu$ that assigns students to schools such that no school has more students than its capacity $q_m$. An assignment mechanism $\phi$ is a function that inputs each school’s capacity $q_m$, students’ reported preferences $Q_i$, and schools’ priorities $F_m$, then outputs a matching $\mu$. We denote the reported preferences of students other than $i$ as $Q_{-i}$. Then, the outcome of mechanism $\phi$ when the reported preferences are $(Q_i, Q_{-i})$ is $\phi(Q_i, Q_{-i})$, with $\phi_i(Q_i, Q_{-i})$ denoting the school that student $i$ matches to under $\phi(Q_i, Q_{-i})$.

An assignment mechanism may have a number of desirable properties. One property is strategy-proofness. A mechanism is **strategy-proof** if truthful revelation of preferences is the weakly dominant strategy for each participant. Strategy-proofness is popular in real-world applications of school choice for a number of reasons. First, it reduces the cost of participating in the school choice problem for all participants by eliminating the need to search for the optimal strategy. Second, it does not punish naive participants who truthfully reveal their preferences. Third, it levels the playing field between those with differential costs to strategizing. Since school choice is often implemented to give students from a disadvantaged background the opportunity to go to better schools, this equity quality is particularly important to policymakers.

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5 Note this is different from a student’s true preferences $P_i$. 
Assignment mechanisms may also be stable. In the school choice context, stability is generally limited to the elimination of justifiable envy. A matching $\mu$ is stable if no student $i$ prefers school $m$ over the school $i$ is assigned to while having a higher priority at $m$ than a student who currently fills a slot at $m$. If the matching is not stable, then student $i$ is justifiably envious of the student taking a slot at the school student $i$ prefers when that school also prioritizes student $i$. A mechanism is stable if it always produces a stable matching. Justifiable envy was cited by the Boston school board as a potential source of lawsuits (Abdulkadiroglu et al., 2006). The elimination of such a risk and the perception of "fairness" make stability a desirable property for policymakers.

A third potential property of assignment mechanisms is Pareto efficiency. A matching $\mu$ is Pareto efficient if there does not exist a matching $\nu$ that all students weakly prefer to $\mu$ and at least one student strictly prefers $\nu$ to $\mu$. When any equilibrium outcome under a mechanism is Pareto efficient, that mechanism is Pareto efficient. A Pareto efficient mechanism is desirable because each student is weakly better off in a Pareto efficient matching than in any other matching. Policymakers desire efficiency because it increases the aggregate gains of all students. Note that Pareto efficiency is often defined with respect to reported preferences as opposed to true preferences, which is how a Pareto efficient mechanism may be less efficient in practice than a mechanism that is not considered Pareto efficient.

2.1 Deferred Acceptance

The DA mechanism is strategy-proof and stable. DA produces a matching that Pareto dominates all other stable matchings, but it is not a Pareto efficient mechanism. The DA mechanism solves the school choice problem by implementing the actions dictated by the students’ reported preferences, the schools’ exogenously-determined priorities, and the tiebreaker. The mechanism proceeds as follows:

- Each student submits an application to their first ranked school (according to their reported preferences).
- Each school rejects the lowest priority students in excess of its capacity and holds the remaining student applications with higher priority.
- Students rejected in the first round apply to their second ranked school.
- Each school considers the new applications together with the applications of the students on hold from the first round. The school rejects the lowest priority students in excess of its capacity and holds the remaining student applications with higher priority.
- Each student rejected in the previous round applies to their next ranked school.
- Each school considers the new applications together with the applications of the students on hold from the last round. The school rejects the lowest priority students in excess of its capacity and holds the remaining student applications with higher priority.

The algorithm terminates when no students are rejected in a round.

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6 More generally, stability is defined as the combination of (i) the elimination of justified envy, (ii) non-wastefulness and (iii) individual rationality. The two latter requirements are automatically satisfied in our experiment.
2.2 Immediate Acceptance

The IA mechanism is neither strategy-proof nor stable. It produces a Pareto efficient matching when all students truthfully reveal their preferences or when they play another Bayesian-Nash equilibrium. The IA mechanism solves the school choice problem by implementing the actions dictated by the students' reported preferences, the schools' exogenously-determined priorities, and the tiebreaker. The mechanism proceeds as follows:

- Each student submits an application to their first ranked school (according to their reported preferences).
- Each school accepts the highest priority students until its available seats are filled. All other students are rejected.
- Each school who still has seats remaining from the first round accepts applications from students who rank the school second.
- Each school accepts students with the highest priority in the second round until its available seats are filled. All other students are rejected.
- In the $i^{th}$ round, each school who still has seats remaining from the $i-1$ round accepts applications from students who rank the school $i^{th}$.
- Each school accepts students with highest priority in the $i^{th}$ round until its available seats are filled. All other students are rejected.

The algorithm terminates when all students have seats at schools.

3 Experimental Design

We replicate Chen and Sönmez's (2006) school choice game to test the effect of strategy advice on participants' strategy choices in the DA and IA mechanisms. The setup is relatively complex compared to other laboratory experiments and aims to mimic applications of the mechanism. We implement a $2 \times 2$ experimental design: an advice treatment and a no advice treatment for each mechanism. The no advice treatment is an exact replication of Chen and Sönmez's experiment and the advice treatment is the replication plus advice.

3.1 The Game

Students compete for one of 36 slots at seven schools in a one-shot game. Based on the students' reported preferences over schools, the schools' priorities over students, and the tiebreaker, the mechanism assigns one student to each slot. No students are unmatched. Student preferences are determined by the monetary payoff to the participant. The payoffs range from $19 for assignment to the student's first-ranked school to $5 for assignment to their last-ranked school. We use the designed environment from Chen and Sönmez, but increase the payoffs by $3. Table 1 shows a sample payoff matrix.

Schools have two priority levels: in-district and not in-district. Students are assigned to a school's district according to Table 2. Schools A and B each have three slots and schools C, D, E, F, and G each have six slots, matching the number of students in each district. Since there are only two priority levels, ties are broken by a random draw of bingo balls after students submit their selections.

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7The designed environment, as opposed to the random environment, correlates preferences to school proximity (lexicographically) and school quality (proxied by smaller quotas).
Table 1: Sample Payoff Matrix

<table>
<thead>
<tr>
<th>Slot Received at School</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff (in $)</td>
<td>12</td>
<td>19</td>
<td>5</td>
<td>16</td>
<td>14</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2: District School Assignments

<table>
<thead>
<tr>
<th>Students</th>
<th>1-3</th>
<th>4-6</th>
<th>7-12</th>
<th>13-18</th>
<th>19-24</th>
<th>25-30</th>
<th>30-36</th>
</tr>
</thead>
<tbody>
<tr>
<td>District School</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
</tr>
</tbody>
</table>

Students are informed of their payoffs, each school’s quota and priorities, and the mechanism’s algorithm in the instructions. The only information students do not have are the preferences of other students and the outcome of the tiebreaker.

3.2 Implementation

We implemented the experiment at Georgia State University’s Experimental Economics Laboratory in November 2016. We recruited participants for eight sessions that took place over two days. There were two sessions per treatment and exactly 36 participants participated in each session. Table 3 shows the number of participants in each experimental treatment.

Table 3: Number of Participants by Treatment

<table>
<thead>
<tr>
<th></th>
<th>DA</th>
<th>IA</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Advice</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>Advice</td>
<td>72</td>
<td>72</td>
</tr>
</tbody>
</table>

We used the instructions verbatim from Chen and Sönmez (2006) (provided in Appendices B and C). The experimenter distributed paper copies of the instructions. Participants filled out paper answer sheets, which the experimenter collected and entered into a Python solver while participants completed a survey about the experiment on lab computers (provided in Appendix D). In one section of the post-experiment survey, participants answered four questions about a logic puzzle. Participants could earn $0.50 for each correct answer for up to $2 total. Participants were paid $20.28 on average at the end of the experiment in cash, including a $5 show up fee.

3.3 The Advice

Participants in advice treatment sessions received strategy advice as an additional page at the end of the instructions packet (Appendices E and F). We frame the advice as the way to "obtain the highest possible payout" and include an illustration of the advice using the example from the instructions. The advice to participants in the DA mechanism is to tell the truth. "In other words, you should rank the schools in the order of your payoffs, from high to low."\(^8\)

We advise participants in the IA mechanism that "you will not necessarily obtain the highest possible payoff" by telling the truth (emphasis included in the advice). The advice recommends one of two strategies. The first strategy suggests listing their district school first. For the remainder of this paper, we call this strategy the "Safe" strategy. The second possible strategy is characterized as riskier and recommends listing their top preference first.

\(^8\)The example in our DA advice has a typo in the last paragraph: "student 3 had applied to Huron" should say Ontario instead. Thank you to an anonymous reviewer for pointing it out.
and district school second. We call this strategy the "Risky" strategy. Note that almost any strategy can be justified by a participant’s preferences and beliefs, so these are only two of many strategies that are undominated. We chose these two strategies for their prominence in real-world applications of assignment mechanisms and in the experimental literature.

4 Strategy Advice Results

Table 4 lists the results of our experiment by treatment, where "NA" is the no advice treatment and "A" is the advice treatment. To interpret these results, first note that "Plays Truthfully" has a slightly different meaning for the DA versus IA mechanisms. In DA, playing truthfully measures how many participants truthfully report their preferences by ordering schools according to their monetary payoff through their district school. In other words, playing truthfully means submitting a ranking of schools from highest payoff to lowest payoff, through the participant’s district school. Participants are guaranteed admission to their district school in DA, so any rankings below the district school are irrelevant to the assignment mechanism. In IA, participants are not guaranteed admission to their district school unless they rank it first; therefore, playing truthfully means truthfully reporting all seven schools.10,11

Table 4 also lists the percentage of participants in IA treatment sessions that play the Safe strategy and the Risky strategy. As described above, participants rank their district school first in the Safe strategy, guaranteeing their assignment. Participants list their top-ranked school first and district school second in the Risky Strategy. Last, we report the number of participants who say in the post-experiment survey that they tried to follow the advice.12 The remainder of this section discusses first the DA results and then the IA results.

<table>
<thead>
<tr>
<th></th>
<th>IA, NA</th>
<th>IA, A</th>
<th>DA, NA</th>
<th>DA, A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plays Truthfully</td>
<td>26%</td>
<td>8%</td>
<td>31%</td>
<td>50%</td>
</tr>
<tr>
<td>IA Safe Strategy</td>
<td>43%</td>
<td>53%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IA Risky Strategy</td>
<td>17%</td>
<td>31%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tries Advice</td>
<td></td>
<td>83%</td>
<td></td>
<td>89%</td>
</tr>
<tr>
<td>Observations</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
</tr>
</tbody>
</table>

4.1 Deferred Acceptance

The weakly dominant strategy in DA is to truthfully reveal preferences. Our strategy advice explains that ordering their schools from highest to lowest payoff leads to the highest

---

9A student is high priority at their district school. No matter where they rank their district school, they are admitted if they apply. There are the same number of slots at each school as there are district residents, so even if all high priority students apply to the district school, each are admitted. If a student ranks their district school first, they are matched to it automatically. If a student ranks their district school third, they are matched to that school if both their first and second choices reject them.

10The only exception is to students whose district school is also their most preferred. In this case, participants need only truthfully report their district school in the top ranking and all of them do. We classify them as following both strategies.

11The percentage of DA strategies that truthfully reveal all seven schools are 17% for the no advice treatment and 43% for the advice treatment.

12Participants chose "Yes" or "No" in response to the question "Did you try to follow the additional advice you were given?"
possible payoff and provides an example to show why that is the case.

**HYPOTHESIS 1:** Participants are more likely to choose the dominant strategy of truth-telling when it is recommended by strategy advice in the DA mechanism.

We find that participants in the DA advice treatment sessions are 19 percentage points more likely to truthfully reveal their preferences than in the DA no advice treatment (50% compared to 31%, respectively). A one-sided test of proportions rejects the null hypothesis that there is no difference in proportions at the 1% significance level (p-value = 0.0087 in a one-sided test).

**RESULT 1:** The truth-telling rate is significantly higher in the advice treatment than in the no advice treatment in the DA mechanism.

This treatment effect of advice is large compared to the low baseline rate of truthful revelation. Fewer than one third of participants truthfully reveal their preferences in the no advice treatment, despite the fact that we do not impose truth-telling beyond the district school. Even when we advise participants that they should tell the truth, only 50% do so. This low figure is not due to a lack of trust in the advice. 89% of participants report trying to follow the advice in the post-experiment survey. The problem appears to be in either understanding or implementing the advice. Of those who say they tried to follow the advice, only 56% successfully do so.

While the treatment effect is large, the proportion of people who truthfully reveal their preferences is still low relative to the truth-telling rates in other school choice laboratory experiments. Exactly half of participants truthfully reveal their preferences in the advice treatment, which is still 22 percentage points lower than in the experiment we replicate where there is no advice (Chen and Sönmez, 2006). Other lab experiments find rates ranging from 44% (Klijn, Pais, and Vorsatz, 2013) to 79% (Basteck and Mantovani, 2018). This result suggests that baseline truth-telling rates are likely heterogeneous across populations.

### 4.2 Immediate Acceptance

There is no dominant strategy in IA. We offer strategy advice that warns participants against telling the truth and suggest one of two heuristic strategies as alternatives. There are two ways in which participants can follow the strategy advice. First, participants can implement one of the two strategies we recommend.

**HYPOTHESIS 2A:** Participants are more likely to choose a heuristic strategy when it is recommended by strategy advice in the IA mechanism.

The second way that participants can follow the advice is to not truthfully reveal their preferences.

**HYPOTHESIS 2B:** Participants are less likely to truthfully reveal their preferences when advised against it by strategy advice in the IA mechanism.

In the IA advice treatment, we find that 10 percentage points more participants (over a base of 43%) choose the Safe strategy and almost double the proportion of participants (14 percentage points on a base of 17%) choose the Risky strategy. A one-sided test of proportions confirms a statistically significant increase in the percentage of Risky strategies at the 5% level (p-value = 0.0249 in a one-sided test), but the difference is not significant at traditional levels for the Safe strategy (p-value = 0.1215 in a one-sided test). The total
number of participants who choose one of the heuristic strategies increases 27 percentage points, from 51% to 78%, in the advice treatment.

RESULT 2A: The proportion of participants who choose a heuristic strategy is significantly higher in the advice treatment than in the no advice treatment. More of the increase is due to inducing participants to choose the Risky strategy than the Safe strategy.

We instruct participants that it may not be in their best interest to tell the truth and almost all participants act in accordance with this advice. Excepting the strategy profiles in which their district school is also their top-ranked school, only two participants truthfully reveal their preferences in the IA Advice sessions. The decrease in truth-telling rates from 26% to 8% is statistically significant at the 1% level (p-value = 0.0001 in a one-sided test).

RESULT 2B: The truth-telling rate is significantly lower in the advice treatment than in the no advice treatment of the IA mechanism.

Participants who report trying to follow the advice are also much more likely to successfully do so in IA versus DA. Slightly fewer participants (83%) report trying to follow advice compared to DA, but 83% of those who try to follow the advice successfully do so by implementing one of the two recommended strategies.

Participants respond to the strategy advice heterogeneously by the ranking (in terms of payoff) of their district school. The increase in the proportion of participants who choose the Risky strategy is entirely due to participants with their district school ranked fourth or lower. Figure 1 graphs the difference in the proportion of participants who choose the Risky strategy in the advice treatment versus the no advice treatment by district school payoff ranking.

On the other hand, only participants with district schools ranked third or fourth contribute to the increase in the percentage of participants who choose the Safe strategy in the advice treatment. Figure 2 graphs the difference in the proportion of participants who choose the Safe strategy in the advice treatment versus the no advice treatment by district school payoff rank. Note that (fortunately) no participants with district school ranked seventh played the Safe strategy in either session.

Compared to Chen and Sönmez (2006), our replication finds a higher rate of truth-telling in the IA mechanism. In our no advice treatment, which replicates Chen and Sönmez’s experiment, we find that almost double (26%) their 14% of participants truthfully reveal their preferences. Again, truth-telling rates and strategic behavior in general may vary across populations.

4.3 Not Following the Advice

The participants who did not implement the recommended strategy may have been rational to do so or the incentives we offered may have been too low to induce participants to care about them. To address both of these concerns, we calculate the expected payoff from playing each of the recommended strategies for those who do not follow the advice. To calculate this expected payoff, as well as the expected payoff to the empirical strategy the participant used, we use recombinant estimation to smooth over two sources of randomness. The first source of randomness is session effects that arise from a participant being in one

---

13There are two of these strategy profiles in each session. Each of these participants chose the dominant strategy for their profile of listing their district school first.

14Note that we cannot calculate something comparable for participants who play one of the recommended strategies because we do not know the counter-factual strategy.
Figure 1: Difference in Proportion of Participants in Advice and No Advice Treatment who Play Risky Strategy by District School Payoff Ranking

Notes: The size of the bubble represents how many participants of each district school rank played the Risky strategy.

session of a treatment rather than another. Second, recombinant estimation averages out a participant’s good or bad luck from a particular tiebreaker.\textsuperscript{15}

We follow the recombinant estimation technique from Mullin and Reiley (2000) to calculate expected payoffs. Define a student profile $\rho_i \forall i = \{1, ..., n\}$, where $n$ is 36 in our experiment, as a list of student $i$’s preference profile and district school. For each of our eight sessions (two sessions each for four treatments), one participant is assigned to each of the 36 student profiles. So, for every student profile $\rho_i$, we observe two strategies per treatment. The recombinant technique address the first source of randomness, session effects, by randomly choosing one of those two strategies for all student profiles except for the participant of interest. Recombinant estimation addresses the second source of randomness, the tiebreaker, by drawing a new tiebreaker for each recombination.

The recombinant estimation proceeds as follows:

1. Fix the strategy $Q_i$ of the participant of interest with student profile $\rho_i$.
2. For each other student profile $\rho_j \forall j \in \{1, ..., n\}/i$, draw an observed strategy $Q_j$ from one of the two treatment sessions.
3. Draw a tiebreaker.
4. Implement the mechanism $\phi$ and record the payoff of the participant of interest $\phi_i(Q_i, Q_{-i})$.
5. Repeat for $r$ recombinations.

\textsuperscript{15}Recall that the participants do not know the result of the tiebreaker until after they have submitted their preferences.
Figure 2: Difference in Proportion of Participants in Advice and No Advice Treatment who Play Safe strategy by District School Payoff Ranking

Notes: The size of the bubble represents how many participants of each district school rank played the Safe strategy.

For example, suppose we fix participant 1’s strategy. Then, we draw one of the strategies played by a participant with student profile 2 from either session one or session two of the treatment. We do the same for student profile 3, ..., through student profile 36. Then, we calculate the payoff to participant 1 by drawing a tiebreaker and implementing the mechanism. We repeat this process for the number of desired recombinations to obtain an estimate of the participant’s expected payoff, given the play of other participants in this experiment.

We list the average difference between the expected payoff from playing each recommended strategy and the observed strategy for each participant who did not choose any of the recommended strategies in Table 5.

As expected, a participant would always be better off playing the dominant strategy of truth-telling in DA. The size of the difference is economically significant. For example, the $1.80 average gains from telling the truth in the no advice treatment is approximately 13% of the total amount a participant could earn. The expected payoff difference increases to $1.92 in the advice treatment.

In stark contrast, participants who chose not to implement one of the heuristic strategies in IA would have had much worse outcomes playing a recommended strategy. For example, the cost of playing the Safe strategy in the IA no advice treatment to a participant who chose not to play either of the heuristic strategies is $2.78, or about 20% of the total dollars.

16Participant payoffs ranged from $5 to $19, for a total of $14 that a participant could earn apart from guaranteed payments.
Table 5: Recombinant Estimation of Difference between Expected Payoffs from Empirical Strategy and Recommended Strategy

<table>
<thead>
<tr>
<th></th>
<th>Risky</th>
<th>Safe</th>
<th>Truthful</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IA, NA</td>
<td>IA, A</td>
<td>IA, NA</td>
</tr>
<tr>
<td>Mean (in $)</td>
<td>-1.32</td>
<td>-2.53</td>
<td>-2.78</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>4.07</td>
<td>2.81</td>
<td>4.26</td>
</tr>
<tr>
<td>Number</td>
<td>33</td>
<td>16</td>
<td>33</td>
</tr>
</tbody>
</table>

Notes: Expected payoff to each strategy is calculated from 10,000 recombinations per participant. We report the standard deviation of expected payoff differences between participants who did not play one of the recommended strategies and the number of participants in each treatment who did not play one of the recommended strategies.

a participant could earn. This result suggests that the participants who chose not to play one of the recommended strategies in IA appear to have been rational in doing so.

5 Mechanism Performance under Sub-optimal Strategies

To evaluate mechanism performance under sub-optimal play, we use the recombinant technique detailed above to estimate the efficiency and stability of DA and IA in the no advice treatments versus the advice treatments. Table 6 lists the recombinant estimation of efficiency for each treatment.

Contrary to theoretical predictions (Miralles, 2009; Abdulkadiroglu et al., 2011; Troyan, 2012), but consistent with other experiments (e.g. Chen and Sönmez, 2006; Calsamiglia et al., 2010), we find that DA is more efficient than IA in our experiment. The difference between the two mechanisms in the no advice treatments ($13.95 to $13.72) is about half the size Chen and Sönmez found ($11.71 to $11.15). In the advice treatment, however, the gap nearly triples ($13.87 to $13.23). While we show that DA is more efficient than IA when participants in both mechanisms choose sub-optimal strategies,17 it is important to note that the differences in efficiency between our treatments are small relative to the difference between efficiency in our experiment and efficiency under optimal strategies.

When we estimate efficiency under universal truth-telling, IA outperforms DA as theory shows it must ($15.55 to $15.07), but efficiency drops considerably for both mechanisms as sub-optimal strategies are added. The larger decrease in IA suggests that sub-optimal strategies have a more pronounced effect on the efficiency of IA compared to DA.

Table 6: Recombinant Estimation of Average Per Capita Payoff

<table>
<thead>
<tr>
<th></th>
<th>IA*</th>
<th>IA, NA</th>
<th>IA, A</th>
<th>DA*</th>
<th>DA, NA</th>
<th>DA, A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>15.55</td>
<td>13.72</td>
<td>13.23</td>
<td>15.07</td>
<td>13.95</td>
<td>13.87</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.11</td>
<td>0.393</td>
<td>0.238</td>
<td>0.24</td>
<td>0.353</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Notes: The mean is calculated from 10,000 recombinations per participant in each treatment for a total of 72,000 total assignments. We report the standard deviation of the average per capita payoff between replications.

A within-mechanism comparison of treatments shows that the greater efficiency of DA is driven more by IA’s loss of efficiency than DA’s efficiency gains from fewer sub-optimal

17Even though we do not know which strategies are sub-optimal in our data, we know that at least one is sub-optimal because the resulting matching is less efficient than the DA matching.
strategies in the advice treatment. In fact, in DA, we observe a small efficiency decline in the advice treatment ($13.95 to $13.87). To the contrary, in IA, the decline in efficiency from the no advice treatment to the advice treatment is nearly $0.50. That is, when more participants are induced to choose a heuristic strategy, the average welfare of participants declined by 3.5% of the total they could earn.

When we turn to stability, DA again outperforms IA. Table 7 lists the average number of blocking pairs from the recombinant estimation— the higher the number of blocking pairs (i.e. the more students who are justifiably envious), the less stable the assignment is. In the no advice treatment, IA has approximately 50% more blocking pairs than DA. Since DA is theoretically stable and IA’s Pareto efficiency is inconsistent with stability (Abdulkadiroğlu and Sönmez, 2003), we expect this result; however, when participants choose sub-optimal strategies, DA could be less stable than IA in practice. Although the DA matchings are not completely stable in our experiment, we find that DA is still more stable than IA in the presence of sub-optimal strategies.

<table>
<thead>
<tr>
<th></th>
<th>IA, NA</th>
<th>IA, A</th>
<th>DA, NA</th>
<th>DA, A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>13.53</td>
<td>15.10</td>
<td>9.42</td>
<td>7.55</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.65</td>
<td>3.52</td>
<td>2.64</td>
<td>2.01</td>
</tr>
</tbody>
</table>

Notes: The mean is calculated from 10,000 recombinations per participant in each treatment for a total of 72,000 total assignments. We report the standard deviation of the average per capita payoff between replications.

Strategy advice has an opposite effect on the stability of DA and IA. While the number of blocking pairs increases by about 1.5 in the IA advice treatment, the number of blocking pairs decreases by approximately two in the DA advice treatment.

In our experiment, we find that DA outperforms IA in welfare measures that average over all participants. Furthermore, our strategy advice exacerbates the difference in performance between the two mechanisms. We turn next to the welfare of individual participants. To evaluate the relative welfare of participants, we must formulate counterfactual strategies since each participant only participates in one mechanism. We do so by developing a new typology to partially order sub-optimal strategies according to their relative dominance in DA.

5.1 A Dominance-Ordered Typology of Strategies in DA

We characterize the dominant strategy in DA as the strategy resulting from understanding three strategic aspects of DA. The first strategic aspect is that a participant’s district school acts as a "safety" school. That is, a participant can never be assigned to a school they rank lower than their district school. A participant who understands this aspect of DA should therefore never rank a school they prefer less than their district school above their district school. Note that the student’s reported preferences $Q_i$ is the strategy, so we refer to them as such in this section.

DISTRICT-CONSISTENT. A strategy $Q_i$ is District-Consistent for student $i$ if $Q_i$ does not rank any school above $i$’s district school that their true preferences $P_i$ rank below $i$’s district school.

18 This is specific to the school choice environment where every district school has enough seats to accept all students in the district.
The second strategic aspect of DA is that a participant can never lose from truthfully ordering the schools they rank above their district school. Note that a participant also cannot lose from truthfully ordering the schools they rank below their district school, but that ordering does not matter strategically.\footnote{As noted earlier, a participant can never be assigned to a school they rank lower than their district school.}

**Ordered.** A strategy $Q_i$ is *Ordered* for student $i$ if the schools $Q_i$ ranks above $i$’s district school are ranked in the same order as in their true preferences $P_i$.

The last strategic aspect in our typology of DA is that, when a student’s strategy is Ordered, the student can never lose from ranking more schools they prefer to their district school above their district school. When their strategy is not Ordered, a participant can also always improve their strategy by ranking one additional school they prefer to their district school *directly above* their district school. Overall, it is always possible for a student to improve a strategy by ranking more schools they prefer to their district school above their district school.

**$x$-Strategy.** If $a_i$ is the number of schools student $i$ prefers to their district school, then for any $x \in \{1, \ldots, a_i\}$, $Q_i$ is an $x$-Strategy if $Q_i$ ranks $x$ schools student $i$ prefers to their district school above $i$’s district school.

We denote District-Consistent strategies as DC and NDC otherwise. Ordered strategies are O and not-Ordered strategies as NO. Then a District-Consistent, Ordered, $x$-Strategy is denoted $Q_i^{DC|O|x}$. The example below illustrates the typology.

**Example 1.** Suppose that $i$’s preferences are $P_i : s_1 \ s_2 \ s_3 \ s_4 \ s_5 \ s_6 \ s_7$, where $i$’s district school $s_4$ is underlined. Then $a_i = 3$ for this $i$ and $P_i$. Some example strategies and their typology are:

- **Strategy** $Q_i^{DC|O|3} : s_1 \ s_2 \ s_3 \ s_4 \ s_7 \ s_6 \ s_5$ is a 3-strategy that is both District-Consistent and Ordered. It is one of $i$’s dominant strategies.
- **Strategy** $Q_i^{DC|NO|3} : s_2 \ s_1 \ s_3 \ s_4 \ s_5 \ s_6 \ s_7$ is a 3-strategy that is District-Consistent, but not Ordered.
- **Strategy** $Q_i^{DC|O|2} : s_1 \ s_3 \ s_4 \ s_5 \ s_2 \ s_6 \ s_7$ is a 2-strategy that is both District-Consistent and Ordered.
- **Strategy** $Q_i^{NDC|N|O|2} : s_3 \ s_5 \ s_1 \ s_4 \ s_5 \ s_6 \ s_7$ is a 2-strategy that is neither District-Consistent nor Ordered.
- **Strategy** $Q_i^{NDC|N|O|2} : s_3 \ s_5 \ s_4 \ s_5 \ s_2 \ s_1 \ s_6 \ s_7$ is a 2-strategy that is Ordered but not District-Consistent.

Observe that all DC|O|$a_i$ are dominant strategies. DC|O|0 are District-first strategies in which a participant ranks their district school first. Strategies DC|O|$x$ for $x \in \{1, \ldots, a_i - 1\}$ exhibit what the literature calls *district school bias* (Chen and Sönmez, 2006). Note that a 1-strategy or a 0-strategy that is also District-Consistent is necessarily Ordered. As a consequence, there are no DC|NO|1 and DC|NO|0 strategies (the lowest $x$ for which DC|NO|$x$ strategies exists is $x = 2$).

\footnote{As noted earlier, a participant can never be assigned to a school they rank lower than their district school.}

19
Table 8: Distribution of Strategies Played in DA

<table>
<thead>
<tr>
<th>Strategy types</th>
<th>Advice</th>
<th>No-Advice</th>
<th>Random</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominant strategies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC</td>
<td>O</td>
<td>aᵢ</td>
<td>50%</td>
</tr>
<tr>
<td>District-first strategies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC</td>
<td>O</td>
<td>0</td>
<td>15.3%</td>
</tr>
<tr>
<td>Other DC</td>
<td>O</td>
<td>x strategies</td>
<td></td>
</tr>
<tr>
<td>{DC</td>
<td>O</td>
<td>1, ..., DC</td>
<td>O</td>
</tr>
<tr>
<td>DC</td>
<td>NO</td>
<td>x strategies</td>
<td></td>
</tr>
<tr>
<td>{DC</td>
<td>NO</td>
<td>2, ..., DC</td>
<td>NO</td>
</tr>
<tr>
<td>Other strategies</td>
<td>9.7%</td>
<td>11%</td>
<td>48%</td>
</tr>
</tbody>
</table>

Notes: The percentages in the last column are the sample averages over 10,000 draws of randomly drawn reported preferences and random student profiles (among the 36 profiles in our experimental design).

The strategies in our typology are connected through dominance-based partial orderings. For any two sets of strategies X and Y, let X → Y indicate that all strategies in X weakly dominate all strategies in Y. Let X ↔ Y indicate that, for every strategy y ∈ Y, there exists a strategy in X that weakly dominates Y. Observe that X → Y and Y ↔ Z imply X → Z, but X ↔ Y and Y → Z only imply X ↔ Z (and not X → Z).

Proposition 1 (Dominance relations between sets of strategies in DA). For any student i and any preferences Pᵢ, the following binary relations hold:

\[
\begin{align*}
\text{DC|O|aᵢ} & \rightarrow \text{DC|O|}(aᵢ - 1) \rightarrow \ldots \rightarrow \text{DC|O|2} \rightarrow \text{DC|O|1} \rightarrow \text{DC|O|0} \\
\text{DC|NO|aᵢ} & \rightarrow \text{DC|NO|}(aᵢ - 1) \rightarrow \ldots \rightarrow \text{DC|NO|2}
\end{align*}
\]

While sub-optimal play has long been documented in DA, this typology is the first attempt (to our knowledge) to partially order the non-dominant strategies. Classifying strategies by this typology can reveal which strategic aspects of DA participants do, and do not, understand. To this end, Table 8 compares the distribution of strategies played in our advice and no advice DA treatments using this typology. The last column of Table 8 shows the distribution of strategies based on randomly drawn reported preferences and a randomly drawn student type from the 36 profiles in our experimental design.

Table 8 shows that, although participants often fail to choose the dominant strategy, about 90% of the participants understand enough about the mechanism to play a District-Consistent strategy in both treatments. Participants choosing randomly would only choose District-Consistent strategies about half of the time. Furthermore, the majority (about 75%) of participants choose an Ordered strategy. These distributions suggest that 1) participants in our experiment are not playing randomly and 2) most understand the
"safety" school and truthful ordering aspects of DA. Most participants who choose a sub-optimal strategy appear to fail to understand the x-Strategy aspect of DA. In other words, they exhibit district school bias.

Our experiment was not designed to study the impact of strategy advice on the distribution of sub-optimal strategies in DA. It is informative, however, to note that the advice does not substantially increase the number of participants who play District-Consistent strategies. Rather, the advice seems to steer participants who would have played a District-Consistent strategy anyways, particularly a District-first strategy, toward the dominant strategy of truth-telling.

The remainder of this section considers the individual welfare of participants who choose a heuristic strategy in IA. We build a series of increasingly irrational, though District-Consistent, counter-factual strategies in DA for the participant to play using the dominance-ordered typology in this section. Then, we use recombinant estimation to estimate expected payoffs to each strategy in their respective mechanisms. We first consider the participants who chose the Safe strategy in IA.

5.2 Safe Strategy

A participant who chooses the Safe strategy in IA is always assigned to their district school. In DA, the same participant is assured a seat at a school they like at least as well as their district school by reporting a District-Consistent strategy, regardless of whether the strategy is Ordered \( o \in \{O, NO\} \). If, in addition, the participant ranks at least one school they prefer to their district school above their district school, then there is a positive probability that they are assigned to a school they like strictly better than their district school in DA. We denote the weak preference relationship for student \( i \) as \( R_i \).

**Proposition 2.** (i) For any preferences \( P_i \), any District-Consistent strategy \( Q_i^{\text{DC}|x} \), any Safe strategy \( Q_i^{\text{Safe}} \), any strategies played by students \( j \in \{1, \ldots, n\}/i \), \( Q_{-i} \), and any priority profile that is drawn with positive probability,

\[
DA(Q_i^{\text{DC}|x}, Q_{-i}) R_i IA(Q_i^{\text{Safe}}, Q_{-i}).
\]

(ii) If, in addition, \( x \neq 0 \), then there exists \( Q_i^{\text{DC}|x} \) such that for any priority profile that is drawn with positive probability,

\[
DA(Q_i^{\text{DC}|x}, Q_{-i}) P_i IA(Q_i^{\text{Safe}}, Q_{-i}).
\]

Proposition 2 says that any student playing the Safe strategy in IA can be no worse-off in DA as long as they play a District-Consistent strategy, which around 90% of our participants do (see Table 8). In addition, if the student plays a District-Consistent strategy with at least one school ranked above their district school, they could be strictly better-off in DA. How much and how often participants are strictly better-off under DA compared to IA depends on (a) the preferences reported by other students \( (Q_{-i}) \), and (b) the "level of rationality" of the strategy they implement in DA.\(^{21}\) We use recombinant estimation to empirically estimate this welfare gain with our experimental data.

Quantifying Proposition 2 provides a measure of the welfare gain to a participant who chooses the Safe strategy in IA of participating in the DA mechanism instead. We use

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\(^{20}\)This assumption seems reasonable since approximately 90% of participants play District-Consistent strategy.

\(^{21}\)Suppose that \( A \) and \( B \) are two sets of District-Consistent strategies for student \( i \), and \( A \rightarrow B \). By definition, for any strategy in \( Q_i^B \in B \), there exists a strategy \( Q_i^A \in A \) that weakly dominates \( Q_i^B \). That is, student \( i \) is better-off more often in DA compared to IA when playing \( Q_i^A \) than when playing \( Q_i^B \) (and \( i \) is never worse-off under \( Q_i^A \) than under \( Q_i^B \)).
the same recombinant estimation technique described above to determine a participant’s expected payoff for each possible District-Consistent strategy. Specifically, for each participant who chose $Q_i^{Safe}$ in the IA Advice treatment, we implement the following procedure:

1. Set $Q_i = Q_i^{DC|o|x}$ for $o \in \{O,NO\}$ and $x \in \{1, \ldots, a_i\}$.
2. For each other participant profile $Q_{-i}$, draw one of the observed strategies from the DA Advice treatment.\footnote{Out of two, since there are two sessions for each treatment. See Appendix G for results using the No Advice treatments.}
3. Draw a tie breaker.
4. Implement the DA mechanism and record the payoff of the participant of interest $DA_i(Q_i, Q_{-i})$.

We repeat this procedure 50,000 times for each $x,o$ combination. Note that we do not need to estimate the participant’s payoff from playing the Safe strategy in IA since it will always be the payoff from their district school.

Figures 3 and 4 show the average welfare gain for each Ordered and Not-Ordered strategy by the participant’s district school rank. The x-axis tracks the rank of the district school in the counter-factual DA strategy. For example, for a participant whose district school is ranked fourth (the green starred line), the first point on the x-axis corresponds to the welfare difference between playing a counter-factual strategy with their district school ranked second in DA and playing the Safe strategy in IA. The second point corresponds to a counter-factual strategy with their district school ranked third and the third point to the dominant strategy. Participants with lower ranked district schools have more points on the graph because there are more possible District-Consistent strategies. There are fewer points on the Not-Ordered graph because District-Consistent 1-strategies are necessarily ordered.

The graphs show that participants who chose to play the Safe strategy in IA would have been unequivocally better off in DA. Note that all points are above the x-axis, meaning that all welfare gains are positive. We also find empirical evidence for Proposition 1 by observing the increasing welfare gains from DC|O|1 or DC|NO|2 to DC|O|a_i. Note also that the largest welfare gains are to those participants with district schools ranked lower in their true preferences. In other words, the most disadvantaged participants in the game have the most to gain from switching to any District-Consistent strategy in the DA mechanism from the Safe strategy in the IA mechanism.

5.3 Risky Strategy

Theoretical results are weaker for the Risky strategy. In principle, a participant could be better-off playing the Risky strategy in IA than playing any District-Consistent strategy in DA, including a dominant strategy. It is also possible for a participant to be better-off under a District-Consistent strategy in DA than under the Risky strategy in IA. Comparisons between the District-Consistent strategy in DA and the Risky strategy in IA are entirely dependent on other participants’ reported preferences.

The only systematic advantage of District-Consistent strategies in DA over the Risky strategy in IA is in terms of the worst-case scenario: a District-Consistent strategy in DA guarantees a lower-bound on the participant’s assignment that is at least as good as the lower-bound under the Risky strategy in IA. When the participant’s district school is not their least or most preferred school, this statement can be strengthened to say that the lower-bound is strictly higher under any District-Consistent strategy in DA compared to the
Figure 3: Expected Payoff Difference between District-Consistent Ordered Strategy in DA and Safe Strategy in IA

Notes: Each line traces the estimated welfare change to participants with the same district school rank who chose the Safe strategy in an IA session of playing a District-Consistent Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.

lower-bound under the Risky strategy in IA. In other words, the lower-bound assignment for the Risky strategy in IA is the participant’s least preferred school. The lower-bound assignment for a District-Consistent strategy in DA is the district school. So long as these two do not intersect and the assignment is not trivial, the lower-bound in a District-Consistent strategy in DA is strictly higher than the lower-bound in the Risky strategy in IA.

Proposition 3. (i) For any preferences $P_i$, any District-Consistent strategy $Q_{i}^{DC\lvert o\rvert x}$, any Risky strategy $Q_{i}^{Risky}$, and any priority profile that is drawn with positive probability,

$$\min_{P_i} \{DA_i(Q_{i}^{DC\lvert o\rvert x}, Q_{-i}) \text{ for all } Q_{-i} \} \ P_i \ \min \{IA_i(Q_{i}^{Risky}, Q_{-i}) \text{ for all } Q_{-i} \}.$$ 

(ii) If in addition, i’s district school is not their first- or least-preferred school, then

$$\min_{P_i} \{DA_i(Q_{i}^{DC\lvert o\rvert x}, Q_{-i}) \text{ for all } Q_{-i} \} \ P_i \ \min \{IA_i(Q_{i}^{Risky}, Q_{-i}) \text{ for all } Q_{-i} \}.$$ 

As the name suggests, participants take a chance in choosing the Risky strategy. By ranking a school above their district school, a participant risks losing their priority at their district school if their district school fills its quota in the first round. Then, participants who choose the Risky strategy may be assigned to a school they like less than their district school, which is impossible in DA for students who play a District-Consistent strategy in
Figure 4: Expected Payoff Difference between District-Consistent Not-Ordered Strategy in DA and Safe Strategy in IA

Notes: Each line traces the estimated welfare change to participants with the same district school rank who chose the Safe strategy in an IA session of playing a District-Consistent Not-Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.

DA. It is even plausible — and we observe it in our simulations — for participants implementing the Risky strategy to be matched to their least preferred school when their district school is ranked considerably higher in their true preferences. At the same time, participants who have well-calibrated beliefs about other participants’ strategies could obtain much higher payoffs than would otherwise be plausible in DA, particularly when other participants implement the Safe strategy in IA. We again turn to recombinant estimation to empirically evaluate whether participants who choose the Risky strategy in IA would be better off on average than by playing a District-Consistent strategy in the DA mechanism.

We calculate the welfare change of participants who chose the Risky strategy in an IA treatment of instead implementing a District-Consistent strategy in DA. We replicate the recombinant estimation of expected payoffs to playing a series of increasingly irrational strategies in DA from the previous subsection. Then, since the participant’s payoff to playing the Risky strategy is not known ex-ante, we use recombinant estimation again to estimate the participant’s expected payoff in IA. We repeat the procedure 50,000 times for each strategy.

Figures 5 and 6 show that most participants would be better off playing any District-Consistent strategy in DA than playing the Risky strategy in IA. Participants whose district school is ranked third in their true preferences and choose an Ordered strategy stand out as the exception, but the size of the difference is small compared to the payoff increases that other participants expect to receive. We, again, observe that the greatest welfare gains of switching to the DA mechanism are realized by participants with low-ranked district
Figure 5: Expected Payoff Difference between District-Consistent Ordered Strategy in DA and Risky Strategy in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose the Risky strategy in an IA session of playing a District-Consistent Ordered $x$-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.

5.4 Other Strategies

Almost any strategy can be justified in the IA mechanism by a participant’s beliefs about the strategies of other players. Participants who choose strategies that correspond neither to our Risky or Safe strategy are not guaranteed their district school because they do not rank it first. The same as for the Risky strategy, the minimum payoff is lower in IA compared to DA for all participants whose district school is not ranked last. On the other hand, these other strategies may yield higher payoffs than playing even the dominant strategy in DA depending on the strategy choices of the other players and the tie breaker. We use recombinant estimation to evaluate how the “other” strategies we observe in our IA sessions compare to District-Consistent strategies in the DA mechanism.

We calculate the welfare change of participants who choose neither the Risky nor Safe strategy in an IA session of instead implementing increasingly irrational District-Consistent strategies in a DA session. Our recombinant estimation, again, estimates the participant’s expected payoff from their chosen strategy in IA and their expected payoff from playing a series of counter-factual strategies in DA. We repeat the procedure 50,000 times to estimate the welfare difference for each participant.

Figures 7 and 8 show mixed results for participants who choose non-heuristic strategies in IA. For participants with district schools ranked 5th or 6th, ranking more than two
Figure 6: Expected Payoff Difference between District-Consistent Not-Ordered Strategy in DA and Risky Strategy in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose the Risky strategy in an IA session of playing a District-Consistent Not-Ordered Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.

schools above their district school generates higher welfare in the DA mechanism; however, for participants with district schools ranked 4th and 7th, their welfare is near uniformly better playing their chosen strategy in an IA session. These contrary results suggest that, as would be expected, the expected welfare gains from participating in the DA mechanism compared to the IA mechanism is largely dependent on the participant’s chosen strategy. For example, participants who ranked one of their middle-preferred schools first, when that school is also always under-subscribed in the first round of IA, did particularly well relative to other participants with the same district school rank. In summary, these results show that while participants likely to choose a heuristic strategy fare worse in the IA mechanism, disadvantaged participants who play sophisticated strategies in the IA mechanism allows them to obtain higher payoffs compared to even the dominant strategy in DA.

6 Discussion

Our strategy advice decreases the proportion of sub-optimal strategies in DA. The increase in the proportion of participants who choose the dominant strategy in DA is large and statistically significant, but our strategy advice fails to achieve truth-telling rates close to the theoretically optimal 100%. Participants may be sensitive to the phrasing of the strategy advice, how it is communicated, and how trustworthy they find the person giving the advice. Future research integrating insights from behavioral economics and marketing research may shed light on how to produce more compelling strategy advice. On the other
Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose neither the Risky strategy nor the Safe strategy in an IA session of playing a District-Consistent Ordered strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.

hand, the existence of sub-optimal play in the medical residency match (Rees-Jones, 2018) should temper any expectations of designing an information environment that induces optimal play for all participants.

The increase in the proportion of participants who choose one of the recommended heuristic strategies in the IA advice treatment is disconcerting when taken together with the overall welfare loss in that treatment. We cannot say for certain that the participants who were induced to choose one of the heuristic strategies in the advice treatment would have done better to not follow the advice. One, we do not know which participants would have played the heuristic strategy regardless of strategy advice and which were induced by the advice. Two, we do not know the counter-factual strategy the participant would have chosen. We do know that the strategy advice has a negative and significant effect on overall welfare. This result is concerning due to the prevalence of strategy advice like ours in the field.

A more important issue may be that we do not have a replacement for the "bad" advice that is currently pervasive. There is no generally "good" advice since the optimal strategy varies greatly based on participants' beliefs, preferences, and priorities. Moreover, journalists and bloggers are unlikely to stop giving strategy advice even though there is not universally good advice. This problem with strategy advice in IA adds to the evidence we find that favors the performance of DA relative to IA. In our experiment, DA is more efficient and more stable, even when only 31% of participants choose the dominant strategy.
Figure 8: Expected Payoff Difference between District-Consistent Not-Ordered Strategy in DA and Other Strategies in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose neither the Risky strategy nor the Safe strategy in an IA session of playing a District-Consistent Not-Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.

Overall, our results suggest that DA outperforms IA under sub-optimal strategies. In other words, while it may be theoretically possible for IA to outperform DA, we are unlikely to observe such an outcome in real-world applications that resemble the game played in this experiment.

There are important caveats to this conclusion. First, our efficiency analysis is inherently ex post; therefore, we do not address the theoretical result that IA ex-ante Pareto dominates any strategy-proof mechanism (Troyan, 2012). Second, our preferences do not vary in cardinality. Miralles (2009) and Abdulkadiroğlu et al. (2011) argue that IA may be more efficient due to varying levels of preference intensities. Those with higher preference intensities may take more risks when manipulating their preferences and obtain higher payoffs. We cannot address this issue with our current experiment and leave that to future research.

Our analysis of individual welfare shows that almost any participant who chose a heuristic strategy in IA would have preferred to walk into one of our DA sessions as long as they play a District-Consistent strategy (which almost all of our participants do). Participants who would benefit the most from switching to the DA mechanism from the IA mechanism are those who start the game at a disadvantage. These results have important equity implications for policymakers since they often emphasize the expected equity gains from implementing a school choice program. Disadvantaged students who are likely to play heuristic strategies in IA, particularly when these strategies are widely recommended, may
actually be worse off under school choice.

Suppose a student in a disadvantaged area chose the Risky strategy. That student may end up at a worse school, one that is further away and not better, or even unassigned. If they do not have an outside option such as private school, the risk of being unassigned may make the gamble even higher stakes. Disadvantaged students, then, may be even more likely to play the Safe strategy. The Safe strategy, of course, leaves these disadvantaged students at the same school they would attend without school choice. The equity quality of the DA mechanism takes on greater meaning in light of these results.

The welfare gains from switching to DA only extend to some participants who chose neither the Risky nor the Safe strategy in the IA mechanism. Participants who choose sophisticated strategies, especially those at a particular disadvantage in our experiment, fare better playing their chosen strategies in IA. This result qualifies our conclusions for individual welfare in an intuitive way: participants who take advantage of the manipulability of the IA mechanism using a sophisticated strategy and have well-calibrated beliefs about other participants’ strategies fare better in the less equitable mechanism.

References


Appendices

A Proofs

Proof of Proposition 1.

Step 1: DC|O|a_i → DC|O|(a_i - 1) and DC|O|a_i → DC|NO|a_i.

These binary relations follow from the fact that all strategies in DC|O|a_i are dominant strategies.

Step 2: DC|O|k → DC|O|(k - 1) for every k ∈ {2, ..., a_i - 1}.

Take any strategy Q_i ∈ DC|O|(k - 1), and let d be i’s district school. Let S be the set of schools student i ranks above their district school in Q_i. Because (k - 1) < a_i, there exists a school s* that student i prefers to their district school such that s* ∉ S. Let Q_i^∗ be the strategy in DC|O|k for which S^∗ = S ∪ {s*} is the set of schools i ranks above their district school (i.e., Q_i^∗ is constructed from Q_i by “moving” s* from below to above the district school and ranking s* truthfully).

Preferences in (1) to (4) hold for all strict priority profiles that can result from the tiebreaker. Because DA is strategy-proof (Dubins and Friedman, 1981),

\[ DA_i(Q_i, Q_{-i}), \quad \text{for all } Q_{-i}. \]

Then, since Q_i^∗ ranks schools S^∗ ∪ {s*, d} identically to student i’s true preferences P_i,

\[ DA_i(Q_i^∗, Q_{-i}) R_i DA_i(Q_i, Q_{-i}), \quad \text{for all } Q_{-i} \text{ such that } DA_i(Q_i^∗, Q_{-i}), DA_i(Q_i, Q_{-i}) \in S^∗ ∪ \{d\}. \] (2)

But because S is the set of schools student i ranks above their district school in Q_i, and S^∗ the set of schools student i ranks above their district school in Q_i^∗, we have DA_i(Q_i, Q_{-i}), DA_i(Q_i^∗, Q_{-i}) ∈ S^∗ ∪ \{d\} for all Q_{-i}. Hence, (2) implies

\[ DA_i(Q_i^∗, Q_{-i}) R_i DA_i(Q_i, Q_{-i}), \quad \text{for all } Q_{-i}. \] (3)

Lastly, consider any Q^*_{-i} s.t. all students except i and j rank their district schools first. Let student j ≠ i with district school s* rank d above s* in Q_j^∗. Then, all students except i and j are assigned to their district schools and there is one slot open in s* and one slot open in d. Under Q_i, student i is assigned to d since they apply to d before s* and are admitted. Under Q_j^∗, student i is assigned to s* since they apply to s* before d and hold a seat there until student j applies to d and is admitted. Then, we have

\[ DA_i(Q_i^∗, Q_{-i}) = s^* P_i d = DA_i(Q_i, Q_{-i}). \] (4)

Together, (3) and (4) show that Q_i^∗ dominates Q_i. Because Q_i is an arbitrary strategy in DC|O|(k - 1), this completes the proof of this step.

Step 3: DC|NO|k → DC|NO|(k - 1) for every k ∈ {2, ..., a_i}. Take any strategy Q_i ∈ DC|NO|(k - 1), and let d be i’s district school. Let S be the set of schools i ranks above their district school in Q_i. Because (k - 1) < a_i, there exists a school s* that i prefers to their district school such that s* ∉ S. Let Q_i^∗ be the strategy in DC|NO|k for which

(a) \( S^* = S \cup \{s^*\} \) is the set of schools i ranks above their district school,

(b) schools in \( S \) are ranked exactly as in Q_i, and

(c) s* is ranked directly above the district school, i.e., s* Q_i^∗ d, and s Q_i^∗ s* for all s ∈ \( S \) (i.e., Q_i^∗ is constructed from Q_i by “moving” s* from below to right above the district school, and not changing any other rankings compared to Q_i).
Again, (10) holds for all strict priority profiles that can result from the tiebreaker. By construction,

\[ DA_i(Q^*_i, Q_{-i}) = DA_i(Q_i, Q_{-i}), \]

for all \( Q_{-i} \) such that \( DA_i(Q^*_i, Q_{-i}), DA_i(Q_i, Q_{-i}) \in \hat{S} \).

Also, because \( i \) prefers \( s^* \) to \( d \),

\[ DA_i(Q^*_i, Q_{-i}) P_i DA_i(Q_i, Q_{-i}), \]

for all \( Q_{-i} \) such that \( DA_i(Q^*_i, Q_{-i}) = s^* \) and \( DA_i(Q_i, Q_{-i}) = d \).

and

\[ DA_i(Q^*_i, Q_{-i}) = d \implies DA_i(Q_i, Q_{-i}) = d. \] (7)

Together, (5) to (7) imply

\[ DA_i(Q^*_i, Q_{-i}) R_i DA_i(Q_i, Q_{-i}), \quad \text{for all } Q_{-i}. \] (8)

Lastly, consider any \( Q^*_{-i} \) s.t. all students except \( i \) and \( j \) rank their district schools first. Let student \( j \neq i \) with district school \( s^* \) rank \( d \) above \( s^* \) in \( Q^*_i \). Then, all students except \( i \) and \( j \) are assigned to their district schools in round 1 and there is one slot open in \( s^* \) and one slot open in \( d \). Under \( Q_i \), student \( i \) is assigned to \( d \) since they apply to \( d \) before \( s^* \) and are admitted. Under \( Q^*_j \), student \( j \) is assigned to \( s^* \) since they apply to \( s^* \) before \( d \) and are admitted, while student \( j \) is assigned to \( d \). Then, we have

\[ DA_i(Q^*_i, Q^*_{-i}) = s^* \quad P_i \quad d = DA_i(Q_i, Q^*_j). \] (9)

Together, (3) and (4) show that \( Q^*_i \) dominates \( Q_i \). Because \( Q_i \) is an arbitrary strategy in \( DC[O](k - 1) \), this completes the proof of this step.

**Step 4:** \( DC[O](k) \rightarrow DC[NO](k) \) for every \( k \in \{2, \ldots, a_i - 1 \} \). Take any strategy \( Q_i \in DC[NO](k) \), and let \( d \) be \( i's \) district school. Let \( \hat{S} \) be the set of schools \( i \) ranks above their district school in \( Q_i \). Let \( Q^*_i \) be the strategy in \( DC[O](k) \) for which \( \hat{S} \) is the set of schools \( i \) ranks above their district school (i.e., \( Q^*_i \) is constructed from \( Q_i \) by truthfully re-ordering the schools in \( \hat{S} \)).

By the same argument that lead to (3) in Step 2, we have

\[ DA_i(Q^*_i, Q_{-i}) R_i DA_i(Q_i, Q_{-i}), \quad \text{for all } Q_{-i}. \] (10)

Again, (10) holds for all strict priority profiles that can result from the tiebreaker.

Because \( Q_i \) is District-Consistent but not-Ordered, there exists two schools \( \hat{s}, \tilde{s} \in \hat{S} \) such that \( \hat{s} \) \( Q_i \) \( \tilde{s} \) but \( \hat{s} \) \( P_i \) \( \tilde{s} \).

Consider any \( Q^*_{-i} \) in which

(a) every student \( j \neq i \) whose district school is ranked above \( \hat{s} \) in \( Q_i \) ranks their district school first,

(b) every student \( j \neq i \) whose district school is \( \hat{s} \) ranks \( \hat{s} \) first, except for one of these students who ranks \( \hat{s} \) first and

(c) every student \( j \neq i \) whose district school is \( \hat{s} \) ranks \( \hat{s} \) first, except for one of these students who ranks \( d \) first,

(d) every student \( j \neq i \) whose district school is \( d \) ranks \( d \) first.
Let \( h \neq i \) be the student whose district school is \( \tilde{s} \) and who ranks \( \hat{s} \) first. Let \( l \neq i \) be the student whose district school is \( \hat{s} \) and who ranks \( d \) first. In both \( DA(Q_i, Q_{-i}^*) \) and \( DA(Q_i^*, Q_{-i}^*) \), every student but \( i, h, \) and \( l \) is assigned to their district school. This implies that, in both \( DA(Q_i, Q_{-i}^*) \) and \( DA(Q_i^*, Q_{-i}^*) \), student \( i \) is necessarily rejected from any school they apply to other than \( \hat{s}, \tilde{s}, \) and \( d \).

Under both \( DA(Q_i, Q_{-i}^*) \) and \( DA(Q_i^*, Q_{-i}^*) \), \( l \) initially applies to \( d \) and is held at \( d \) as no more than \( q_s \) students initially apply to \( d \) (only students with \( d \) as their district school apply to \( d \) and \( i \) who has \( d \) as their district school does not). Similarly, \( h \) initially applies to \( \tilde{s} \) and is held at \( \tilde{s} \) as no more than \( q_s \) students initially apply to \( \hat{s} \) (only students with \( \hat{s} \) as their district school apply to \( \hat{s} \) and \( l \) who has \( \hat{s} \) as their district school does not). This remains true at least until \( h \) applies to either \( d \) or \( \hat{s} \).

In \( DA(Q_i, Q_{-i}^*) \), after having been rejected from all the schools student \( i \) ranks above \( \hat{s}, i \) applies to \( \tilde{s} \). Because there is a seat available at \( \tilde{s}, i \) is held at \( \tilde{s} \), all students are assigned, and the assignment is therefore final. That is,

\[
DA_i(Q_i, Q_{-i}^*) = \tilde{s}.
\] (11)

In contrast, in \( DA(Q_i^*, Q_{-i}^*) \), after having been rejected from all the schools student \( i \) ranks above \( \hat{s}, i \) applies to \( \tilde{s} \). When \( i \) applies to \( \tilde{s} \), all seats at \( \tilde{s} \) are already occupied, one of which by \( h \) who does not have \( \hat{s} \) as their district school. Thus, \( i \) is held at \( \tilde{s} \) exactly half the time, i.e., when \( i \) is given higher priority than \( h \) by the tie-breaking rule. That is,

\[
DA_i(Q_i^*, Q_{-i}^*) = \tilde{s}, \quad \text{with probability one-half.}
\] (12)

In (11), (12), and (13) together imply \( i \)'s expected utility under \( DA(Q_i^*, Q_{-i}^*) \) is larger than under \( DA(Q_i, Q_{-i}^*) \). Together with the fact that (10) holds for every realization of the tiebreaker, this shows that \( Q_i^* \) dominates \( Q_i \). Because \( Q_i \) is an arbitrary strategy in DC-k, this completes the proof of this step.

**Proof of Proposition 2.**

Let \( Q_i \) be any strategy \( Q_i \in DC|O|x. \) (i) Because \( i \)'s district school \( d \) has as many seats as there are students in \( i \)'s district,

\[
IA_i(Q_i^{safe}, Q_{-i}) = d \quad \text{for all } Q_{-i}.
\] (14)

Also, because \( DA \) is strategy-proof (?),

\[
DA_i(Q_i^{DA}, Q_{-i}) \leq Q_i^{DA} \quad DA_i(Q_i, Q_{-i}), \quad \text{for all } Q_{-i} \text{ and all } Q_i.
\] (15)

In particular,

\[
DA_i(Q_i^{DA}, Q_{-i}) \leq Q_i^{DA} \quad DA_i(Q_i^{safe}, Q_{-i}) = d, \quad \text{for all } Q_{-i}.
\] (16)

But because \( Q_i^{DA} \) is District-Consistent, this implies

\[
DA_i(Q_i^{DA}, Q_{-i}) R_i d = IA_i(Q_i^{safe}, Q_{-i}^{IA}), \quad \text{for all } Q_{-i}^{DA}, Q_{-i}^{IA},
\] (17)

the desired result.

(ii) Let \( s^* \) be the school that is ranked first in \( Q_i^{DA} \). Because \( Q_i^{DA} \) is District-Consistent but is not District-First, \( s^* \) \( P_i \) \( d \). Let \( Q_i^{DA} \) be such that
(a) every student \( j \neq i \) whose district school is different from \( d \) and \( s^* \) ranks their district school first,

(b) all but one student \( j \neq i \) whose district is \( s^* \) ranks \( s^* \) first, with the student who does not rank \( s^* \) first ranking \( d \) first, and

(c) every student \( j \neq i \) whose district school is \( d \) ranks \( d \) first.

By (c) and because \( i \) ranks \( s^* \) first, the student whose district school is \( s^* \) and who ranks \( s^* \) first is assigned to \( d \). Every other student \( j \neq i \) is assigned to their district school. This implies that there is an available seat school \( s^* \) and that \( i \) is assigned to \( s^* \) (as \( i \) ranks \( s^* \) first). But then we have

\[
DA_i(Q_{IA}^{DA}, \bar{Q}_i^{DA}) = s^* \quad \text{for all } Q_{-i}^{IA},
\]

the desired result.

**Proof of Proposition 3.** Let \( f \) be student \( i \)'s most-preferred school, and \( l \) student \( i \)'s least-preferred school.

(i) Let \( \bar{Q}_i^{IA} \) be such that every student \( j \neq i \) ranks their district school first. Thus, \( i \) is rejected from \( f \) when applying to \( f \) in the first round of \( IA \) and others report \( Q_{-i}^{IA} \).

Because only \( q_d - 1 \) students apply to \( d \) in the first round, \( i \) is then assigned to \( d \), that is,

\[
IA_i(Q_{Risky}^{IA}, \bar{Q}_i^{IA}) = d,
\]

which implies

\[
d \in R_i \min_{R_i} \{IA_i(Q_{IA}^{Risky}, Q_{-i}) \text{ for all } Q_{-i}\}.
\]

But by (17), we have

\[
\min_{R_i} \{DA_i(\bar{Q}_i^{DA}, Q_{-i}) \text{ for all } Q_{-i}\} R_i d,
\]

which combined with (20) implies the desired result.

(ii) Let \( \bar{Q}_i^{IA} \) be such that

(a) every student \( j \neq i \) ranks their district school first, except for one student \( j^* \) whose district school is \( l \),

(b) student \( j^* \) ranks school \( d \) first.

By (a), \( i \) is rejected from school \( f \) when applying to \( f \) in the first round. By (a) and (b), all seats are occupied at the end of the first round, except for one seat at school \( l \). Thus,

\[
IA_i(Q_{IA}^{Risky}, \bar{Q}_i^{IA}) = l,
\]

which implies

\[
\min_{R_i} \{IA_i(Q_{IA}^{Risky}, Q_{-i}) \text{ for all } Q_{-i}\} = l.
\]

But by (17), we have

\[
\min_{R_i} \{DA_i(\bar{Q}_i^{DA}, Q_{-i}) \text{ for all } Q_{-i}\} R_i d.
\]

By assumption, \( d \) is not \( i \) least preferred school, i.e., \( d \neq P_i \). Therefore (23) and (24) imply the desired result.
B DA Instructions

Instructions

This is an experiment in the economics of decision making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money. In this experiment, we simulate a procedure to allocate students to schools. The procedure, payment rules, and student allocation method are described below. Do not communicate with each other during the experiment. If you have questions at any point during the experiment, raise your hand and the experimenter will help you.

Procedure

- There are 36 participants in this experiment. You are participant 1.
- In this simulation, 36 school slots are available across seven schools. These schools differ in size, geographic location, specialty, and quality of instruction in each specialty. Each school slot is allocated to one participant. There are three slots each at schools A and B, and six slots each at schools C, D, E, F and G.
- **Your payoff** amount depends on the school slot you hold at the end of the experiment. These amounts reflect the desirability of the school in terms of location, specialty and quality of instruction.

<table>
<thead>
<tr>
<th>Slot received at School:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff to Participant 1 (in dollars)</td>
<td>16</td>
<td>19</td>
<td>12</td>
<td>5</td>
<td>8</td>
<td>14</td>
<td>10</td>
</tr>
</tbody>
</table>

The table is explained as follows:

- You will be paid $16 if you hold a slot at school A at the end of the experiment.
- You will be paid $19 if you hold a slot at school B at the end of the experiment.
- You will be paid $12 if you hold a slot at school C at the end of the experiment.
- You will be paid $5 if you hold a slot at school D at the end of the experiment.
- You will be paid $8 if you hold a slot at school E at the end of the experiment.
- You will be paid $14 if you hold a slot at school F at the end of the experiment.
- You will be paid $10 if you hold a slot at school G at the end of the experiment.

*NOTE* different participants might have different payoff tables. That is, payoff by school might be different for different participants.

- During the experiment, each participant first completes the Decision Sheet by indicating school preferences. The Decision Sheet is the last page of this packet. Note that you need to rank all seven schools in order to indicate your preferences.
- After all participants have indicated their preferences, the experimenter will collect the preferences and start the allocation process.
- Once the allocations are determined, the experimenter will inform each participant of his/her allocation slot and respective payoff.
In this experiment, participants are defined as belonging to the following school districts:

- Participants #1 - #3 live within the school district of school A,
- Participants #4 - #6 live within the school district of school B,
- Participants #7 - #12 live within the school district of school C,
- Participants #13 - #18 live within the school district of school D,
- Participants #19 - #24 live within the school district of school E,
- Participants #25 - #30 live within the school district of school F,
- Participants #31 - #36 live within the school district of school G,

A priority order is determined for each school. Each participant is assigned a slot at the best possible school he/she reported that is consistent with the priority order below.

The priority order for each school is separately determined as follows:

- **High Priority Level:** Participants who live within the school district. Since the number of High priority participants at each school is equal to the school capacity, each High priority participant is guaranteed an assignment which is at least as good as his/her district school based on the ranking indicated in his/her Decision Sheet.
- **Low Priority Level:** Participants who do not live within the school district. The priority among the Low priority students is based on their respective order in a fair lottery. This means each participant has an equal chance of being the first in the line, the second in the line, . . . , as well as the last in the line. To determine this lottery, the experimenter will draw participant’s ID number randomly, one at a time. The sequence of the draw determines the order in the lottery.

Once the priorities are determined, the allocation of school slots is obtained as follows:

- An application to the first ranked school in the Decision Sheet is sent for each participant.
- Throughout the allocation process, a school can hold no more applications than its number of slots.
  - If a school receives more applications than its capacity, then it rejects the students with lowest priority orders. The remaining applications are retained.
- Whenever an applicant is rejected at a school, his/her application is sent to the next highest school he/she reported.
- Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the lowest priority ones in excess of the number of the slots are rejected, while remaining applications are retained.
- The allocation is finalized when no more applications can be rejected. Each participant is assigned a slot at the school that holds his/her application at the end of the process.
An Example:
We will go through a simple example to illustrate how the allocation method works.

**Students and Schools:** In this example, there are six students, 1-6, and four schools, Clair, Erie, Huron, and Ontario.

| Student ID Numbers: 1,2,3,4,5,6 | Schools: Clair, Erie, Huron, Ontario |

**Slots and Residents:** there are two slots at each Clair and Erie, and one slot each at Huron and Ontario. Residents of districts are indicated in the table below.

<table>
<thead>
<tr>
<th>Schools</th>
<th>Slot 1</th>
<th>Slot 2</th>
<th>District Residents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clair</td>
<td></td>
<td></td>
<td>1 2</td>
</tr>
<tr>
<td>Erie</td>
<td></td>
<td></td>
<td>3 4</td>
</tr>
<tr>
<td>Huron</td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Ontario</td>
<td></td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

**Lotteries:** In this example, the lottery produced the following order

1-2-3-4-5-6

**Submitted School Rankings:** The students have submitted the following school rankings

<table>
<thead>
<tr>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>3rd Choice</th>
<th>Last Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>Huron</td>
<td>Clair</td>
<td>Ontario</td>
</tr>
<tr>
<td>Student 2</td>
<td>Huron</td>
<td>Ontario</td>
<td>Clair</td>
</tr>
<tr>
<td>Student 3</td>
<td>Ontario</td>
<td>Clair</td>
<td>Erie</td>
</tr>
<tr>
<td>Student 4</td>
<td>Huron</td>
<td>Clair</td>
<td>Ontario</td>
</tr>
<tr>
<td>Student 5</td>
<td>Ontario</td>
<td>Huron</td>
<td>Clair</td>
</tr>
<tr>
<td>Student 6</td>
<td>Clair</td>
<td>Erie</td>
<td>Ontario</td>
</tr>
</tbody>
</table>

**Priority:** School priorities first depend on whether the school is a district school, and next on the lottery order:

<table>
<thead>
<tr>
<th>Priority order at Clair</th>
<th>Resident</th>
<th>Non-Resident</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>3-4-5-6</td>
<td></td>
</tr>
<tr>
<td>Priority order at Erie</td>
<td>3-4</td>
<td>1-2-5-6</td>
</tr>
<tr>
<td>Priority order at Huron</td>
<td>5</td>
<td>1-2-3-4-6</td>
</tr>
<tr>
<td>Priority order at Ontario</td>
<td>6</td>
<td>1-2-3-4-5</td>
</tr>
</tbody>
</table>

**The allocation method consists of the following steps:**

**Step 1:** Each student applies to his/her first choice: students 1, 2 and 4 apply to Huron, students 3 and 5 apply to Ontario, and student 6 applies to Clair.

- Clair holds the application of student 6.
- Huron holds the application of student 1 and rejects students 2 and 4
- Ontario holds the application of student 3 and rejects student 5

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Applicants} & \text{School} & \text{Hold} & \text{Reject} \\
\hline
6 & \rightarrow & \text{Clair} & \rightarrow \ \boxed{6} \ \□ \\
\hline
1,2,4 & \rightarrow & \text{Huron} & \rightarrow \ 1 \ 2,4 \\
\hline
3,5 & \rightarrow & \text{Ontario} & \rightarrow \ 3 \ 5 \\
\hline
\end{array}
\]

**Step 2:** Each student rejected in Step 1 applies to his/her next choice: student 2 applies to Ontario, student 4 applies to Clair, and student 5 applies to Huron.

- Clair considers the application of student 4 together with the application of student 6, which was on hold. It holds both applications.
- Huron considers the application of student 5 together with the application of student 1, which was on hold. It holds the application of student 5 and rejects student 1.
- Ontario considers the application of student 2 together with the application of student 3, which was on hold. It holds the application of student 2 and rejects student 3.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Hold} & \text{New Applicants} & \text{School} & \text{Hold} & \text{Reject} \\
\hline
\boxed{6} \ □ & 4 & \rightarrow & \text{Clair} & \rightarrow \ \boxed{6} \ 4 \\
\hline
\ □ \ □ & \rightarrow & \text{Erie} & \rightarrow \ □ \ □ \\
\hline
\boxed{1} & 5 & \rightarrow & \text{Huron} & \rightarrow \ 5 \ 1 \\
\hline
3 & 2 & \rightarrow & \text{Ontario} & \rightarrow \ 2 \ 3 \\
\hline
\end{array}
\]

**Step 3:** Each student rejected in Step 2 applies to his/her next choice: Students 1 and 3 apply to Clair.

- Clair considers the applications of students 1 and 3 together with the applications of students 4 and 6, which were on hold. It holds the applications of students 1 and 3 and rejects students 4 and 6.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Hold} & \text{New Applicants} & \text{School} & \text{Hold} & \text{Reject} \\
\hline
\boxed{6} \ 4 & 1,3 & \rightarrow & \text{Clair} & \rightarrow \ 1 \ 3 \ 4,6 \\
\hline
\ □ \ □ & \rightarrow & \text{Erie} & \rightarrow \ □ \ □ \\
\hline
\boxed{5} & \rightarrow & \text{Huron} & \rightarrow \ 5 \\
\hline
2 & \rightarrow & \text{Ontario} & \rightarrow \ 2 \\
\hline
\end{array}
\]

**Step 4:** Each student rejected in Step 3 applies to his/her next choice: Student 4 applies to Ontario and student 6 applies to Erie.
Ontario considers the application of student 4 together with the application of student 2, which was on hold. It holds the application of student 2 and rejects student 4.

Erie holds the application of student 6.

**Step 5:** Each student rejected in Step 4 applies to his/her next choice: student 4 applies to Erie.

- Erie considers the application of student 4 together with the application of student 6, which was on hold. It holds both applications.

No application is rejected at Step 5. Based on this method the final allocations are:

<table>
<thead>
<tr>
<th>Student</th>
<th>1 2 3 4 5 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School</td>
<td>Clair Ontario Clair Erie Huron Erie</td>
</tr>
</tbody>
</table>

You will have 15 minutes to go over the instructions at your own pace, and make your decisions. Feel free to earn as much cash as you can. Are there any questions?
**Decision Sheet**

- Recall: You are participant 1 and you live within the school district of School A.
- Recall: Your payoff amount depends on the school slot you hold at the end of the experiment. Payoff amounts are outlined in the following table.

<table>
<thead>
<tr>
<th>Slot received at School:</th>
<th>A</th>
<th>B</th>
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<th>D</th>
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The table is explained as follows:

- You will be paid $16 if you hold a slot at school A at the end of the experiment.
- You will be paid $19 if you hold a slot at school B at the end of the experiment.
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- You will be paid $5 if you hold a slot at school D at the end of the experiment.
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- You will be paid $14 if you hold a slot at school F at the end of the experiment.
- You will be paid $10 if you hold a slot at school G at the end of the experiment.

Please write down your ranking of the schools (A through G) from your first choice to your last choice. Please rank ALL seven schools.

<table>
<thead>
<tr>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>3rd Choice</th>
<th>4th Choice</th>
<th>5th Choice</th>
<th>6th Choice</th>
<th>7th Choice</th>
</tr>
</thead>
</table>

Please remain seated until the experimenter collects your Decision Sheet.

After the experimenter collects all Decision Sheets, the experimenter will draw ping pong balls from an urn to generate a fair lottery. The lottery, as well as all participants’ rankings will be entered into a computer after the experiment. The experimenter will inform each participant of his/her allocation slot and respective payoff once it is computed.
C IA Instructions

Instructions

This is an experiment in the economics of decision making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money. In this experiment, we simulate a procedure to allocate students to schools. The procedure, payment rules, and student allocation method are described below. Do not communicate with each other during the experiment. If you have questions at any point during the experiment, raise your hand and the experimenter will help you.

Procedure

• There are 36 participants in this experiment. You are participant 1.

• In this simulation, 36 school slots are available across seven schools. These schools differ in size, geographic location, specialty, and quality of instruction in each specialty. Each school slot is allocated to one participant. There are three slots each at schools A and B, and six slots each at schools C, D, E, F and G.

• Your payoff amount depends on the school slot you hold at the end of the experiment. These amounts reflect the desirability of the school in terms of location, specialty and quality of instruction.

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- You will be paid $14 if you hold a slot at school F at the end of the experiment.
- You will be paid $10 if you hold a slot at school G at the end of the experiment.

*NOTE* different participants might have different payoff tables. That is, payoff by school might be different for different participants.

• During the experiment, each participant first completes the Decision Sheet by indicating school preferences. The Decision Sheet is the last page of this packet. Note that you need to rank all seven schools in order to indicate your preferences.

• After all participants have indicated their preferences, the experimenter will collect the preferences and start the allocation process.

• Once the allocations are determined, the experimenter will inform each participant of his/her allocation slot and respective payoff.
In this experiment, participants are defined as belonging to the following school districts:

- Participants #1 - #3 live within the school district of school A,
- Participants #4 - #6 live within the school district of school B,
- Participants #7 - #12 live within the school district of school C,
- Participants #13 - #18 live within the school district of school D,
- Participants #19 - #24 live within the school district of school E,
- Participants #25 - #30 live within the school district of school F,
- Participants #31 - #36 live within the school district of school G,

In addition, for each school, a separate priority order of the students is determined as follows:

- **Highest Priority Level**: Participants who rank the school as their first choice AND who also live within the school district.
- **2nd Priority Level**: Participants who rank the school as their first choice BUT who do not live within the school district.
- **3rd Priority Level**: Participants who rank the school as their second choice AND who also live within the school district.
- **4th Priority Level**: Participants who rank the school as their second choice BUT who do not live within the school district.
- **13th Priority Level**: Participants who rank the school as their seventh choice AND who also live within the school district.
- **Lowest Priority Level**: Participants who rank the school as their seventh choice BUT who do not live within the school district.

The ties between participants at the same priority level are broken using a fair lottery. This means each participant has an equal chance of being the first in the line, the second in the line, ..., as well as the last in the line. To determine this lottery, the experimenter will draw participant’s ID number randomly, one at a time. The sequence of the draw determines the order in the lottery.

Therefore, to determine the priority order of a student for a school:

- The first consideration is how highly the participant ranks the school when indicating his or her preferences,
- The second consideration is whether the participant lives within the school district or not, and
- The last consideration is the order in the lottery.

Once the priorities are determined, slots are allocated in seven rounds.
Round 1.  
   a. An application to the first ranked school in the Decision Sheet is sent for each participant.  
   b. Each school accepts the students with higher priority order until all slots are filled.  
      These students and their assignments are removed from the system. The remaining  
      applications for each respective school are rejected.  

Round 2.  
   a. The rejected applications are sent to the school he/she ranked second when indicating  
      his/her preferences.  
   b. Each school accepts the students with higher priority order until all slots are filled.  
      These students and their assignments are removed from the system. The remaining  
      applications for each respective school are rejected.  

Round 6.  
   a. The application of each participant who is rejected by his/her top five choices is sent to  
      his/her sixth choice.  
   b. If a school still has slots available, then it accepts the students with higher priority order  
      until all slots are filled. The remaining applications are rejected.  

Round 7.  
   Each remaining participant is assigned a slot at his/her last choice.
An Example:
We will go through a simple example to illustrate how the allocation method works.

Students and Schools: In this example, there are six students, 1-6, and four schools, Clair, Erie, Huron and Ontario.

| Student ID Numbers: 1,2,3,4,5,6 | Schools: Clair, Erie, Huron, Ontario |

Slots and Residents: there are two slots at each Clair and Erie, and one slot each at Huron and Ontario. Residents of districts are indicated in the table below.

<table>
<thead>
<tr>
<th>Schools</th>
<th>Slot 1</th>
<th>Slot 2</th>
<th>District Residents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clair</td>
<td>☐</td>
<td>☐</td>
<td>1 2</td>
</tr>
<tr>
<td>Erie</td>
<td>☐</td>
<td>☐</td>
<td>3 4</td>
</tr>
<tr>
<td>Huron</td>
<td>☐</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Ontario</td>
<td>☐</td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

Lotteries: In this example, the lottery produced the following order

1-2-3-4-5-6

Submitted School Rankings: The students have submitted the following school rankings

<table>
<thead>
<tr>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>3rd Choice</th>
<th>Last Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>Huron</td>
<td>Clair</td>
<td>Ontario</td>
</tr>
<tr>
<td>Student 2</td>
<td>Huron</td>
<td>Ontario</td>
<td>Clair</td>
</tr>
<tr>
<td>Student 3</td>
<td>Ontario</td>
<td>Clair</td>
<td>Erie</td>
</tr>
<tr>
<td>Student 4</td>
<td>Huron</td>
<td>Clair</td>
<td>Ontario</td>
</tr>
<tr>
<td>Student 5</td>
<td>Ontario</td>
<td>Huron</td>
<td>Clair</td>
</tr>
<tr>
<td>Student 6</td>
<td>Clair</td>
<td>Erie</td>
<td>Ontario</td>
</tr>
</tbody>
</table>

Priority: School priorities depend on: (1) how highly the student ranks the school, (2) whether the school is a district school, and (3) the lottery order. Clair: Student 6 ranks Clair first. Students 1, 3 and 4 rank Clair second; among them, student 1 lives within the Clair school district. Students 2 and 5 rank Clair third. Using the lottery order to break ties, the priority order for Clair is 6,1,3,4,2,5.

<table>
<thead>
<tr>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>3rd Choice</th>
<th>4th Choice</th>
<th>5th Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td>3,4</td>
<td>2,5</td>
<td>None</td>
</tr>
<tr>
<td>Resident</td>
<td>Non-Resident</td>
<td>Non-Resident</td>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>

Erie: Student 6 ranks Erie second. Student 3 ranks Erie third. Students 1, 2, 4 and 5 rank Erie fourth; among them student 4 lives within the Erie school district. Using the lottery order to break ties, the priority for Erie is 6-3-4-1-2-5.
Huron: Students 1, 2 and 4 rank Huron first. Student 5 ranks Huron second. Students 3 and 6 rank Huron fourth.

Using the lottery at Huron in order to break ties, the priority for Huron is 1-2-4-5-3-6.

<table>
<thead>
<tr>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>3rd Choice</th>
<th>4th Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1,2,5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Resident</td>
</tr>
</tbody>
</table>

Ontario: Students 3 and 5 rank Ontario first. Student 2 ranks Ontario second. Students 1, 4 and 6 rank Ontario third; among them student 6 lives within the Ontario school district.

Using the lottery at Ontario order to break ties, the priority for Ontario is 3-5-2-6-1-4.

<table>
<thead>
<tr>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>3rd Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,5</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,4</td>
</tr>
<tr>
<td></td>
<td>Non-Resident</td>
<td>Resident</td>
</tr>
</tbody>
</table>

The allocation method consists of the following rounds.

Round 1: Each student applies to his/her first choice: students 1, 2 and 4 apply to Huron, students 3 and 5 apply to Ontario, and student 6 applies to Clair.

- Clair accepts student 6.
- Huron accepts student 1 and rejects students 2 and 4.
- Ontario accepts student 3 and rejects student 5.

<table>
<thead>
<tr>
<th>Applicants</th>
<th>School</th>
<th>Accept</th>
<th>Reject</th>
<th>Slot 1</th>
<th>Slot 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>→ Clair</td>
<td>→ 6</td>
<td></td>
<td>[6]</td>
<td>□</td>
</tr>
<tr>
<td>→ Erie</td>
<td>→ 6</td>
<td></td>
<td>□</td>
<td>□</td>
<td></td>
</tr>
<tr>
<td>1,2,4</td>
<td>→ Huron</td>
<td>→ 1</td>
<td>2,4</td>
<td>[1]</td>
<td></td>
</tr>
<tr>
<td>3,5</td>
<td>→ Ontario</td>
<td>→ 3</td>
<td>5</td>
<td>[3]</td>
<td></td>
</tr>
</tbody>
</table>

Accepted students are removed from the subsequent process.

Round 2: Each student who is rejected in Round 1 then applies to his/her second choice:
Student 2 applies to Ontario, student 4 applies to Clair, and student 5 applies to Huron.

- No slot is left at Ontario, so it rejects student 2.
- Clair accepts student 4 for its last slot.
- No slot is left at Huron, so it rejects student 5.
Round 3: Each student who is rejected in Round 1-2 then applies to his/her third choice.
Students 2 and 5 apply to Clair.

- No slot is left at Clair, so it rejects student 2 and 5.

Round 4: Each remaining student is assigned a slot at his/her last choice:

- Student 2 and 5 receive a slot at Erie.

on this method the final allocations are:

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School</td>
<td>Huron</td>
<td>Erie</td>
<td>Ontario</td>
<td>Clair</td>
<td>Erie</td>
<td>Clair</td>
</tr>
</tbody>
</table>

You will have 15 minutes to go over the instructions at your own pace, and make your decisions. Feel free to earn as much cash as you can. Are there any questions?
Decision Sheet

- Recall: You are participant 1 and you live within the school district of School A.

- Recall: Your payoff amount depends on the school slot you hold at the end of the experiment. Payoff amounts are outlined in the following table.

<table>
<thead>
<tr>
<th>Slot received at School:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff to Participant 1 (in dollars)</td>
<td>16</td>
<td>19</td>
<td>12</td>
<td>5</td>
<td>8</td>
<td>14</td>
<td>10</td>
</tr>
</tbody>
</table>

The table is explained as follows:

- You will be paid $16 if you hold a slot at school A at the end of the experiment.
- You will be paid $19 if you hold a slot at school B at the end of the experiment.
- You will be paid $12 if you hold a slot at school C at the end of the experiment.
- You will be paid $5 if you hold a slot at school D at the end of the experiment.
- You will be paid $8 if you hold a slot at school E at the end of the experiment.
- You will be paid $14 if you hold a slot at school F at the end of the experiment.
- You will be paid $10 if you hold a slot at school G at the end of the experiment.

Please write down your ranking of the schools (A through G) from your first choice to your last choice. Please rank ALL seven schools.

| 1st Choice | 2nd Choice | 3rd Choice | 4th Choice | 5th Choice | 6th Choice | 7th Choice |

Please remain seated until the experimenter collects your Decision Sheet.

After the experimenter collects all Decision Sheets, the experimenter will draw ping pong balls from an urn to generate a fair lottery. The lottery, as well as all participants’ rankings will be entered into a computer after the experiment. The experimenter will inform each participant of his/her allocation slot and respective payoff once it is computed.
D  Post-experiment Survey

1. Did you try to follow the advice on how to rank the schools that you were given?
   - Yes
   - No
   - Other (Please Specify)

2. How clear was the advice on how to rank the schools that you were given?
   - Very unclear
   - Unclear
   - Clear
   - Very clear

3. How good was the advice on how to rank the schools that you were given?
   - Very bad
   - Bad
   - Good
   - Very good

4. How did you decide how to rank the schools on the decision sheet?

5. How clear were the rules that would determine which school you would hold a slot at?
   - Very unclear
   - Unclear
   - Clear
   - Very clear

6. Was there anything about the rules that you did not understand?

7. Generally speaking, would you say that most people can be trusted or that you can’t be too careful in dealing with people?
   - Most people can be trusted
   - You can’t be too careful

8. How many experiments in this lab or a similar lab have you participated in?

9. Did you have any prior experience with procedures in which students rank schools and are assigned a slot to a school, either in an experiment, in your own life, or in classes?
   - No
   - Yes, I participated in an experiment involving a similar problem
   - Yes, I have been exposed to a similar procedure in my own life
   - Yes, I have heard of these procedures in one of my classes
10. In the past, have you ever been lied to or deceived in some way in an experiment?
   • Yes
   • No
   • Other (Please Specify)

11. Do you have any additional remarks or comments about this experiment?

The next 4 questions refer to the below scenario. For each question you answer correctly, you will receive $0.50 for up to $2.00 total.

Four people H, J, K, and L have apartments in the same four-story building. Each person lives on a different floor, from the first floor up to the fourth floor. The following restrictions apply:

• Either H or K lives on the first floor.
• J lives on the floor directly below L.

1. Which of the following could be a list of the four people in order from the first floor up to the fourth floor?
   • H,K,L,J
   • H,J,K,L
   • J,L,K,H
   • K,H,J,L
   • K,J,H,L

2. Which of the following statements CANNOT be true?
   • H lives on the first floor.
   • H lives on the second floor.
   • J lives on the second floor.
   • K lives on the third floor.
   • L lives on the fourth floor.

3. If K lives on the second floor, all of the following statements are true EXCEPT:
   • H does not live on the third floor
   • H does not live on the fourth floor
   • J does not live on the third floor
   • J does not live on the first floor
   • L does not live on the third floor

4. If L lives on the third floor, which of the following statements must be true?
   • H does not live on the first floor.
• J does not live on the first floor.
• J does not live on the second floor.
• K does not live on the first floor.
• K does not live on the fourth floor.

Demographics

1. Gender
   • Male
   • Female
   • Other (Please Specify)

2. Ethnicity
   • American Indian
   • Hispanic or Latino
   • Asian
   • Black or African-American
   • Non-hispanic White
   • Pacific Islander
   • Multiple or Mixed
   • Other (Please Specify)

3. Age

4. Marital Status
   • Never Married (Single)
   • Married
   • Living Together, Partners
   • Separated
   • Divorced
   • Widowed

5. Number of children

6. Age of youngest child (if any)

7. Age of oldest child (if any)

8. Student Status
   • Full-time
   • Part-time
   • Other (Please Specify)
E  DA Advice

Advice on how to rank the schools

Whatever ranking other participants report, you will obtain the highest possible payoff by reporting first the school for which you have the highest payoff, second the school for which you have the second highest payoff, and so on. In other words, you should rank the schools in the order of your payoffs, from high to low.

This follows from the fact that your priority at the different schools does not depend on the ranking of schools that you report. Whatever ranking you report, your priority at the different schools remains the same.

If you are rejected from the first school you apply to, the allocation method always allows you to keep your priority at your later schools.

Consider for instance student 2 in the example we gave you. In Step 1, student 2 applies to school Huron and is rejected because student 1 also applies to Huron, and student 1 has a higher priority than student 2 at Huron.

In Step 2, student 2 applies to school Ontario.

Because student 2 has a higher priority at school Ontario than student 3, student 3 is rejected and student 2 is retained at Ontario, even though student 3 had applied to Huron in a prior step and been retained. There was no loss to student 2 from ranking Huron higher than Ontario.

F  IA Advice

Advice on how to rank the schools

Be careful about the ranking you report: you will not necessarily obtain the highest possible payoff by reporting first the school for which you have the highest payoff, second the school for which you have the second highest payoff, and so on. In other words, it is not necessarily best for you to rank the schools in the order of your payoffs, from high to low.

One possible strategy is to rank first the school within the district of which you live.

A second strategy is to rank first the school for which you have the highest payoff, and rank the school within the district of which you live second.

The second strategy is riskier, but offers you a higher chance of holding a seat at the school for which you have the highest payoff.

The reason it is not necessarily best to rank schools in the order of your payoff is that your priority at different schools depends on the ranking of schools that you report.

For example, at any school you do not rank first, your priority at that school is lower than the priority of any other participant who would rank that school first. This is true even if you live within the school district of that school and the other participant does not.

Consider for example student 2 in the example we gave you. In Step 1, student 2 applies to school Huron and is rejected because student 1 also applies to Huron and has a higher priority than student 2 at Huron.

In Step 2, student 2 applies to school Ontario.

Even though student 2 has a higher lottery at school Ontario than student 3, student 2 is rejected from Ontario and student 3 remains at Ontario because student 3 applied to Ontario in an earlier round than student 2.
G Individual Welfare Results for Participants in No Advice Sessions

Figure 9: Expected Payoff Difference between District-Consistent Ordered Strategy in DA and the Safe Strategy in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose the Safe strategy in an IA session of playing a District-Consistent Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.
Figure 10: Expected Payoff Difference between District-Consistent Not-Ordered Strategy in DA and the Safe Strategy in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose the Safe strategy in an IA session of playing a District-Consistent Not-Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.
Figure 11: Expected Payoff Difference between District-Consistent Ordered Strategy in DA and the Risky Strategy in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose the Risky strategy in an IA session of playing a District-Consistent Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.
Figure 12: Expected Payoff Difference between District-Consistent Not-Ordered Strategy in DA and the Risky Strategy in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose the Risky strategy in an IA session of playing a District-Consistent Not-Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.
Figure 13: Expected Payoff Difference between District-Consistent Ordered Strategy in DA and Other Strategies in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose neither the Risky strategy nor the Safe strategy in an IA session of playing a District-Consistent Ordered x-Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.
Figure 14: Expected Payoff Difference between District-Consistent Not-Ordered Strategy in DA and Other Strategies in IA

Notes: Each line traces the estimated payoff difference to participants with the same district school rank who chose neither the Risky strategy nor the Safe strategy in an IA session of playing a District-Consistent Not-Ordered Strategy in DA. The x-axis tracks the rank of the district school in the counter-factual strategy.